

circuit

An electrical circuit consists of 3 parts.

- Energy sources such as battery or generator
- Load or sink such as lamp or motor
- connecting wires

Active and passive elements.

Energy sources (current or voltage sources) are active elements, capable of delivering power to some external device.

Passive elements are those which are capable only of receiving power.

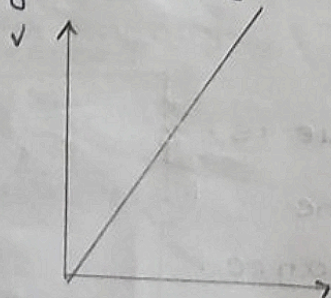
Bilateral and unilateral.

In the bilateral element the voltage current relation is same for current flowing in either direction.

In a unilateral element has different relation between voltage and current for the 2 possible direction of current.

linear and nonlinear elements.

An element is said to be linear if its voltage-current characteristics is all times a straight line through the origin.



A linear element satisfies the principle of superposition. An element which does not satisfy the principle of superposition is known as a non-linear element.

Lumped and distributed elements.

Lumped elements are those elements which are very small in size and in which simultaneously action takes place for any given cause at the same instant of time.

eg: capacitor, resistor, inductor, transformer etc.

Distributed elements are non electrically separable for analytical purpose.

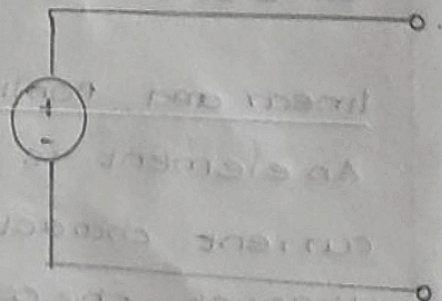
Energy source.

voltage source

current source.

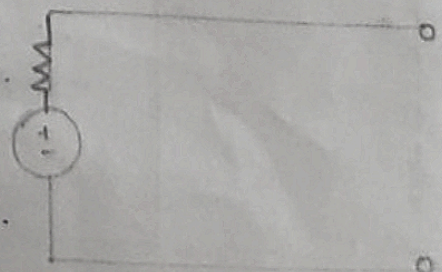
Ideal

It is a constant force and it is completely independent of the current through its terminal.



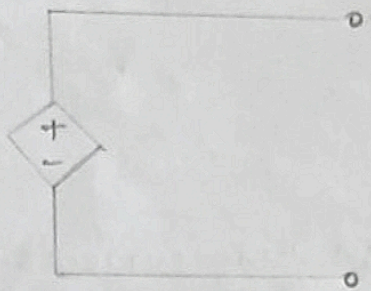
Practical source.

A small resistance value is present in circuit all the time. Source are practical source.

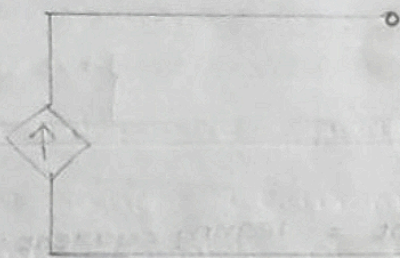


Dependent source.

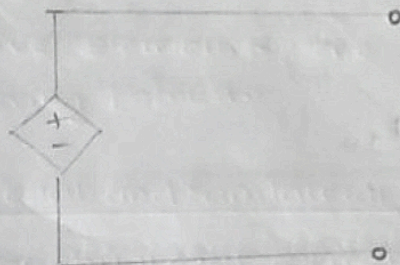
1. Voltage control voltage source.



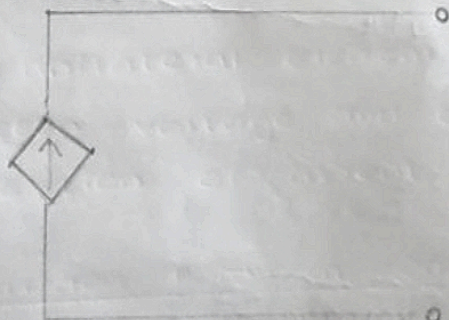
2. Voltage control current source.



3. Current control voltage source.



4. Current control current source.



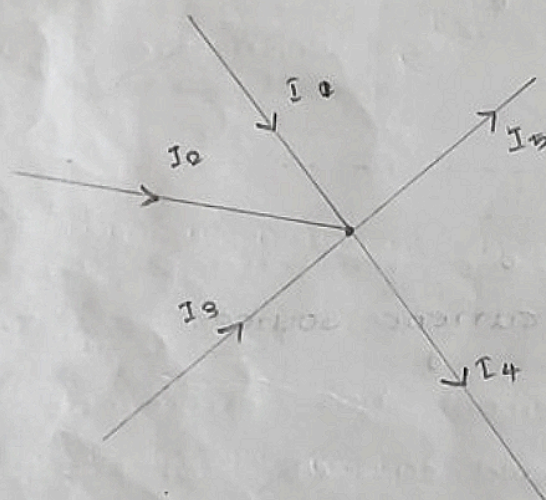
In dependent sources the voltage or current is not fixed but it is dependent on the voltage or current existing at some other location in the circuit.

3/9/2022

KCL

Tuesday

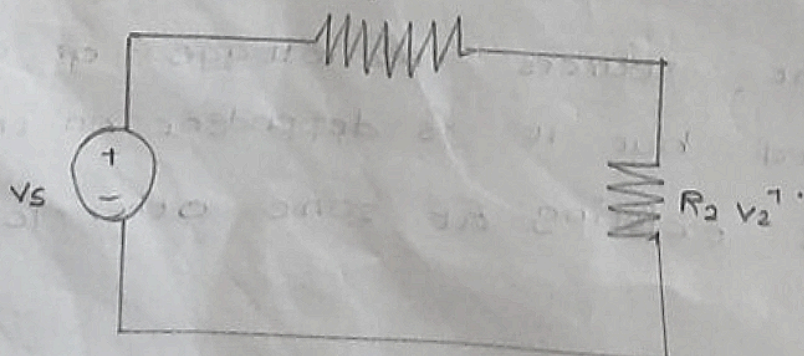
At a node entering current = leaving current.



$$I_1 + I_2 + I_3 = I_4 + I_5$$

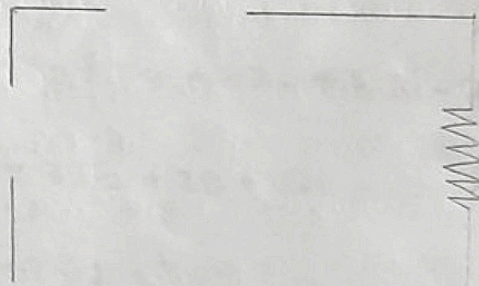
KVL

Sum of
in a loop or mesh, input voltage or supplied voltage =
Sum of voltage drop.



$$V_S = V_1 + V_2$$

Voltage Division Rule



$$V_1 = \frac{\text{Total voltage} \times \text{Resistance}}{\text{Total resistance}}$$

OR

$$V_1 = \frac{V_s \times R_1}{R_1 + R_2} \quad \text{or} \quad V = \frac{V \times R_1}{R_1 + R_2}$$

$$V_2 = \frac{V_s \times R_2}{R_1 + R_2}$$

Current Division Rule

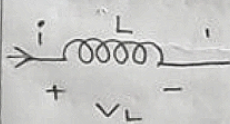
$$I_1 = \frac{I \times R_2}{R_1 + R_2}$$

OR

$$\text{Total current} \times \frac{\text{Opp. resistance}}{R_1 + R_2}$$

$$I_2 = \frac{I \times R_1}{R_1 + R_2}$$

Inductor



$$V_L = L \frac{di}{dt}$$

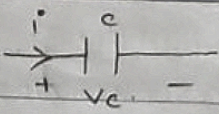
$$\frac{I \times R_1}{R_1 + R_2} \quad V_2 = \frac{V_L \times R_1}{R_1 + R_2}$$

$$V_L = \frac{V_2 \times R_1}{R_1 + R_2}$$

$$V_L = \frac{V_2 \times R_2}{R_1 + R_2}$$

$$V_2 = \frac{V_L \times R_2}{R_1 + R_2}$$

Capacitor

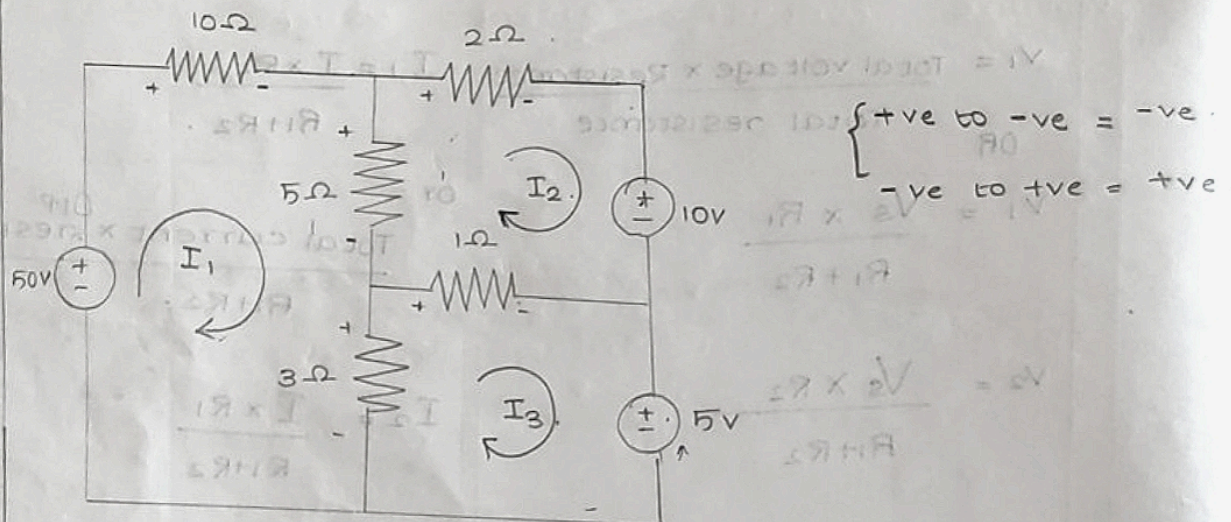


$$V_C = \frac{1}{C} \int i dt$$

$$V_1 = \frac{V_2 \times R_1}{R_1 + R_2}$$

Mesh analysis .

1. count the no. of mesh
2. Apply KVL for each mesh.
3. solve the eqns, then we get loop currents.



Determine mesh current I_1 in the circuit.

No. of mesh = 3

Apply KVL in 1st mesh.

$$50 - 10I_1 - 5(I_1 - I_2) - 3(I_1 - I_3) = 0$$

$$50 - 10I_1 - 5I_1 + 5I_2 - 3I_1 + 3I_3 = 0$$

$$50 - 18I_1 + 5I_2 + 3I_3 = 0$$

$$18I_1 - 5I_2 - 3I_3 = 50 \quad \text{--- (1)}$$

Apply KVL in 2nd mesh

$$-10 - 1(I_2 - I_3) - 5(I_2 - I_1) - 2(I_2) = 0$$

$$-10 - I_2 + I_3 - 5I_2 + 5I_1 - 2I_2 = 0$$

$$-10 - 8I_2 + I_3 + 5I_1 = 0$$

$$-8I_2 + I_3 + 5I_1 = 10 \quad \text{--- (2)}$$

Apply KVL in 3rd mesh.

$$-5 - 3(I_3 - I_1) - 1(I_3 - I_2) = 0$$

$$-5 - 3I_3 + 3I_1 - I_3 + I_2 = 0$$

$$-5 - 4I_3 + 3I_1 + I_2 = 0$$

$$-4I_3 + 3I_1 + I_2 = 5 \quad \text{--- (3)}$$

$$15I_1 - 5I_2 - 3I_3 = 50 \quad I_1 = 8 \quad I_2 = -5 \quad I_3 = -3 \quad = 60$$

$$-8I_2 + I_3 + 5I_1 = 10 \quad I_1 = 5 \quad I_2 = -8 \quad I_3 = 1 \quad = 10$$

$$-4I_3 + 3I_1 + I_2 = 5 \quad I_1 = 3 \quad I_2 = 1 \quad I_3 = -4 \quad = 5$$

$$I_1 = 3.30$$

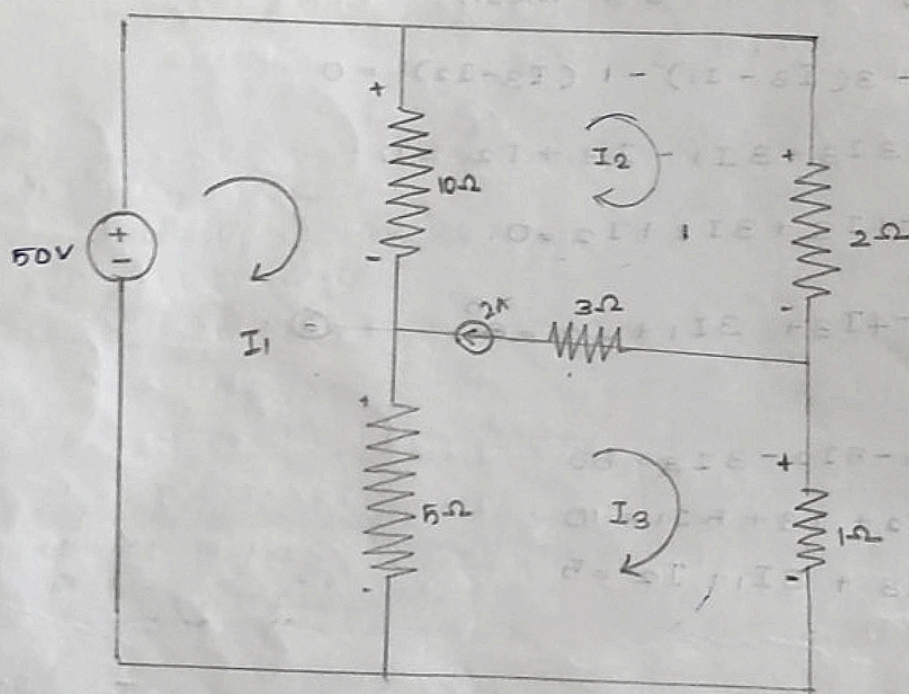
$$I_2 = 0.99$$

$$I_3 = 1.47$$

15-09-22
Thursday

Super Mesh Analysis

- Suppose any of the branches in the network has a current source. Then it is slightly difficult to apply mesh analysis straight forward. To overcome the difficulty is by applying the Supermesh technique. Here we have to choose the kind of Supermesh. A Supermesh is constituted by two adjacent loops that have a common current source.



- Apply KVL in loop 1

$$50 - 10(I_1 - I_2) - 5(I_1 - I_3) = 0$$

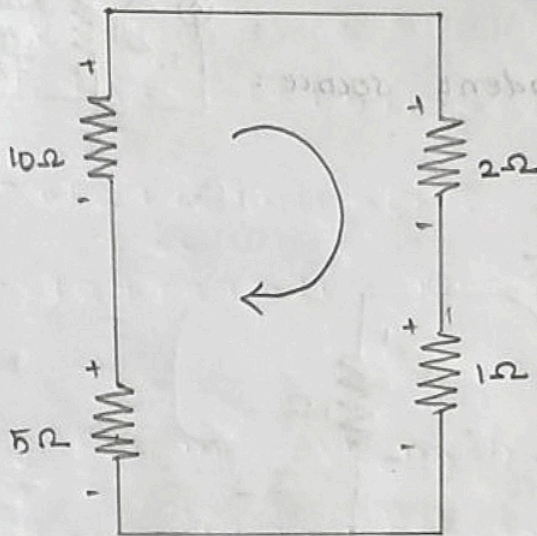
$$50 - 10I_1 + 10I_2 - 5I_1 + 5I_3 = 0$$

$$50 - 15I_1 + 10I_2 + 5I_3 = 0 \quad \text{--- (1)}$$

$$15I_1 - 10I_2 - 5I_3 = 50 \quad \text{--- (2)}$$

consider supermesh and apply KVL

(10Ω , 2Ω , 1Ω , and 5Ω).



$$-10(I_2 - I_1) - 2I_2 - 1I_3 - 5(I_3 - I_1) = 0$$

$$-10I_2 + 10I_1 - 2I_2 - I_3 - 5I_3 + 5I_1 = 0$$

$$10I_1 + 5I_1 - 10I_2 - 2I_2 - I_3 - 5I_3 = 0$$

$$15I_1 - 12I_2 - 6I_3 = 0 \quad \text{--- (2)}$$

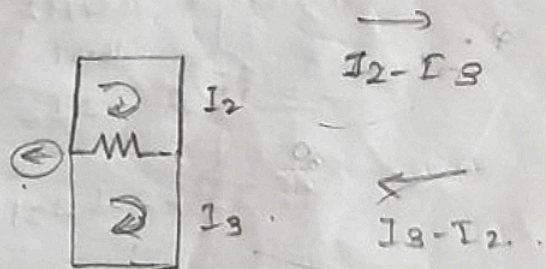
consider branch containing 1A source.

$$I_2 - I_3 = 2 \quad \text{--- (3)}$$

$$I_1 = 20$$

$$I_2 = 17.33$$

$$I_3 = 15.33$$

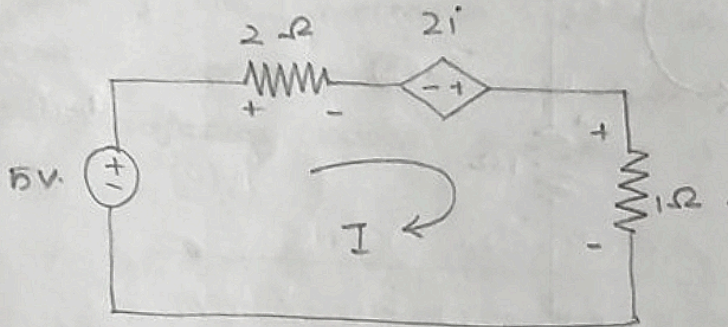
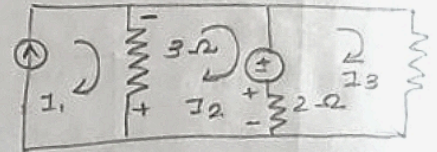


16/9/2022

Friday

Write the mesh eqns for the circuit and find the values of I_1 , I_2 and I_3 .

Mesh analysis - Dependent source:



Apply KVL

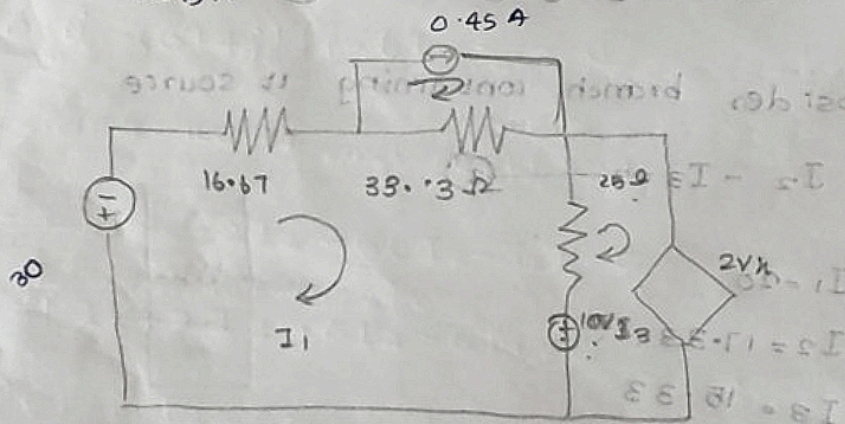
$$5 - 2i + 2i - i = 0$$

$$5 - i = 0$$

$$i = 5A$$

0.45 A

Q.



Apply KVL in loop 1.

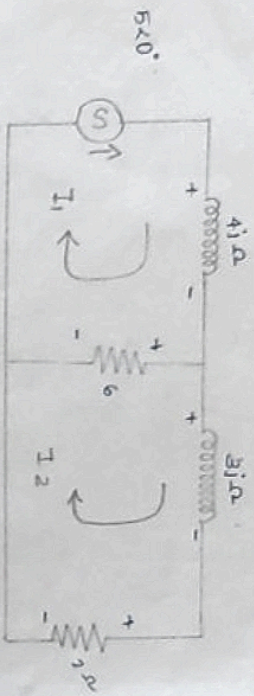
$$-30 - 16.67 I_1 - 33.3 (I_1 - I_2) - 25 (I_1 - I_3) - 10 = 0$$

$$-30 - 16.67 I_1 - 33.3 I_1 + 33.3 I_2 - 25 I_1 + 25 I_3 - 10 = 0$$

$$-40 - 74.97 I_1 +$$

Ac Mesh analysis

Qn. Write the mesh current eqns in the given circuit and determine the currents.



consider loop 1 and apply KVL

$$v_{<0>} - 4j I_1 - 6(I_1 - I_2) = 0$$

$$v_{<0>} = 4j I_1 + 6I_1 - 6I_2 = 0$$

$$v_{<0>} = (6 + 4j) I_1 - 6I_2 \quad \text{--- (1)}$$

consider loop 2 and apply KVL

$$-3j I_2 - 2I_2 - 6(I_2 - I_1) = 0$$

$$3j I_2 + 2I_2 + 6I_2 - 6I_1 = 0$$

$$(6 + 3j) I_2 - 6I_1 = 0 \quad \text{--- (2)}$$

Reduce eqn 2

$$(8 + 3j) I_2 = 6I_1$$

$$\left(\frac{8 + 3j}{6} \right) I_2 = I_1 \quad \text{--- (3)}$$

Sub eqn (3) in (1)

$$(6 + 4j) \left(\frac{8 + 3j}{6} \right) I_2 - 6I_2 = v_{<0>}$$

$$\left(6 + \frac{25}{3}j \right) I_2 - 6I_2 = v_{<0>}$$

$$6I_2 + \frac{25}{3}j I_2 - 6I_2 = v_{<0>}$$

$$\frac{25}{3}j I_2 = v_{<0>}$$

$$I_2 = \frac{v_{<0>}}{25/3j}$$

$$I_2 = -\frac{3}{5}j$$

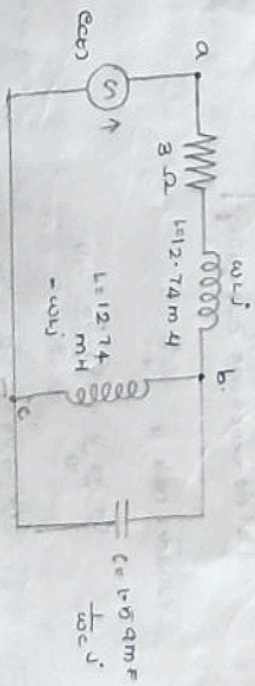
$$I_2 = -0.6j$$

$$I_1 = \left(\frac{8 + 3j}{6} \right) (-0.6j)$$

$$= (0.9 - 0.8j) \text{ A}$$

1. Hw Qn: Tutorial

determine the voltage v_{ab} and v_{bc} in the given network by loop analysis



where $v_{<0>} = 100 \cos(314t + 45^\circ)$

$$100 < 45^\circ$$

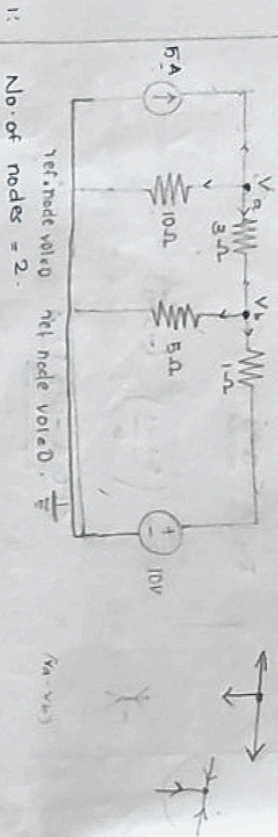
$$L \rightarrow X_L = \omega L$$

$$C \rightarrow X_C = \frac{1}{\omega C}$$

Nodal Analysis

- 1: calculate the no of node
- 2: apply KCL in each node and find eqns.
- 3: solve eqns and then get voltage at each node.

1. Qn: write the node voltage eqns. & determine the currents in each branch.



1: No. of nodes = 2.

Va & Vb are node voltages.

2: Apply KCL

[all elements are outgoing]

consider node a

$$-5 + \frac{V_a - 0}{10} + \frac{V_a - V_b}{3} = 0$$

$$\frac{V_a}{10} + \frac{V_a - V_b}{3} = 5$$

consider node b

$$\frac{V_b - V_a}{3} + \frac{V_b - 0}{1} + \frac{V_b - 0}{5} = 0$$

-5 because elements are outgoing

19-04-2022

$$\frac{V_b - V_a}{3} + \frac{V_b - 0}{1} + \frac{V_b - 0}{5} = 0$$

$$\frac{V_b - V_a}{3} + V_b - 0 + \frac{V_b}{5} = 0$$

$$V_b + \frac{V_b}{3} + \frac{V_b}{5} - \frac{V_a}{3} = 0$$

$$V_b + \frac{V_b}{3} + \frac{V_b}{5} - \frac{V_a}{3} = 0$$

$$\frac{23V_b - V_a}{15} = 0 \quad \text{--- (2)}$$

$$V_a = 19.8 \text{ V} \quad \& \quad V_b = 10.8 \text{ V}$$

Nodal Analysis

$$\text{Current through } 10\Omega = \frac{V_a - 0}{10} = \frac{19.8}{10}$$

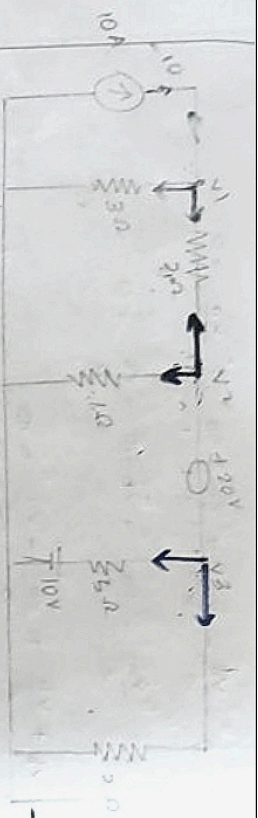
$$\text{Current through } 3\Omega = \frac{V_a - V_b}{3} = \frac{19.8 - 10.8}{3} = 3 \text{ A}$$

$$\text{Current through } 5\Omega = \frac{V_b - 0}{5} = \frac{10.8}{5} = 2.16$$

$$\text{Current through } 1\Omega = \frac{V_b - 0}{1} = \frac{10.8 - 10}{1} = 0.8 \text{ A}$$

Super Node:

Suppose any of the branches in the network has a voltage source then it is slightly difficult to apply nodal analysis. To overcome this we use Super node technique. In this method, the two adjacent nodes that are connected by a voltage source are reduced to a single node and then the eqns. are formed by applying Kirchhoff's current law.



Here we have 3 nodes $\rightarrow V_1, V_2$ and V_3 voltage source (20V) is between V_2 and V_3 .
consider node 1 apply KCL

$$-10 + \frac{V_1 - V_2}{2} + \frac{V_1 - 0}{3} = 0$$

$$\frac{V_1}{2} - \frac{V_2}{2} + \frac{V_1}{3} = 10 \rightarrow (1)$$

$$\frac{5V_1}{6} - \frac{V_2}{2} = 10 \rightarrow (1)$$

consider node 2 and 3 apply KCL

$$\frac{V_2 - V_1}{2} + \frac{V_2 - 0}{1} + \frac{V_3 - 0}{5} + \frac{V_3 - 0}{2} = 0$$

$$\frac{V_2 - V_1}{2} + \frac{V_2}{1} + \frac{V_3}{5} - \frac{10}{5} + 10 \frac{V_3}{2} = 0$$

$$\frac{3V_2}{2} - \frac{V_1}{2} + \frac{7V_3}{10} = \frac{10}{5}$$

$$\frac{3V_2}{2} - \frac{V_1}{2} + \frac{7V_3}{10} = 2$$

Solving (1), (2), (3)

$$V_1 = 18.9$$

$$V_2 = 11.5$$

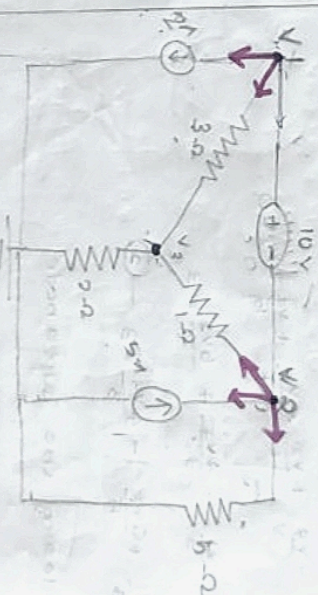
$$V_3 = -8.42$$

Consider the super node
 $V_2 - V_3 = 20V$

current through 5Ω

$$\frac{V_3 - 0}{5} = \frac{-8.42 - 0}{5} = -1.684A$$

Qn2: Find the power delivered by 5A current source in the circuit



consider node 3

$$\frac{V_3 - V_1}{3} + \frac{V_3 - 0}{2} + \frac{V_3 - V_2}{5} = 0$$

$$\frac{V_3 - V_1}{3} + \frac{V_3}{2} + \frac{V_3 - V_2}{5} = 0$$

$$\frac{3V_3 - 2V_1}{6} + \frac{V_3}{2} + \frac{V_3 - V_2}{5} = 0$$

$$\frac{V_3 - V_1}{3} + \frac{V_3}{2} + \frac{V_3 - V_2}{5} = 0$$

$$\frac{-V_1}{3} - \frac{V_2}{5} + \frac{V_3}{2} + \frac{V_3}{5} + \frac{V_3}{5} = 0$$

$$\frac{-V_1}{3} - \frac{V_2}{5} + \frac{5V_3 + 3V_3 + V_3}{15} = 0$$

$$\frac{-V_1}{3} - \frac{V_2}{5} + \frac{9V_3}{15} = 0$$

$$\frac{-V_1}{3} - \frac{V_2}{5} + \frac{3V_3}{5} = 0$$

$$-\frac{V_1}{3} + -V_2 + \frac{11V_3}{6} = 0 \quad \text{--- (1)}$$

consider node (1) and (2).

$$2 + \frac{V_1 - V_3}{8} + \frac{V_2 - V_3}{1} - 5 + \frac{V_2 - 0}{5} = 0 \quad \text{--- (2)}$$

$$\frac{V_1}{8} - \frac{V_3}{8} + \frac{V_2}{1} - \frac{V_3}{1} + \frac{V_2}{5} - 3 = 0.$$

$$\text{cons: } \frac{V_1}{8} - \frac{4V_3}{8} + \frac{6V_2}{5} = 3 \quad \text{--- (3)}$$

$$\frac{V_1}{8} + \frac{6V_2}{5} - \frac{4V_3}{8} = 3 \quad \text{--- (3)}$$

considering the supernode.

$$V_1 - V_2 = 10 \quad \text{--- (3)}$$

Solving (1), (3) and (3).

$$V_1 = 13.7$$

$$V_2 = 3.7$$

$$V_3 = 4.5$$

Power:

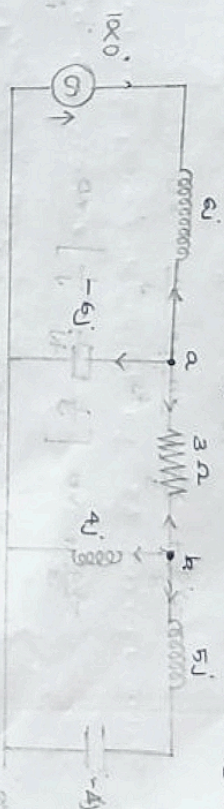
$$P = VI$$

$$= 3.7 \times 5$$

$$= 18.5$$

Nodal Analysis in AC.

Determine V_a and V_b in given circuit. using nodal analysis.



consider node a and apply KCL.

$$\frac{V_a - 10\angle 0^\circ}{6j} + \frac{V_a - 0}{-6j} + \frac{V_a - V_b}{3} = 0$$

$$\frac{V_a}{6j} - \frac{10\angle 0^\circ}{6j} + \frac{V_a}{-6j} + \frac{V_a - V_b}{3} = 0$$

$$\frac{6V_a - 60}{36} + \frac{V_a}{-6} + \frac{3V_a - 3V_b}{36} = 0$$

$$\frac{V_a}{6j} - \frac{10\angle 0^\circ}{6j} + \frac{V_a}{-6j} + \frac{V_a - V_b}{3} = 0$$

$$\frac{-10\angle 0^\circ}{6j} + \frac{V_a}{3} - \frac{V_b}{3} = 0$$

$$\frac{V_a - V_b}{3} = \frac{10\angle 0^\circ}{6j}$$

$$\frac{V_a - V_b}{3} = \frac{-5\angle 90^\circ}{3} \quad \text{--- (1)}$$

consider node b and apply KCL

$$\frac{V_b - V_a}{3} + \frac{V_b - 0}{4j} + \frac{V_b - 0}{5j - 4j} = 0$$

$$= 5j - 4j$$

$$\frac{v_b - v_a}{3} + \frac{v_b}{4j} + \frac{v_b}{j} = 0$$

$$-\frac{v_a}{3} + \frac{v_b}{3} + \frac{v_b}{4j} + \frac{v_b}{j} = 0$$

$$-\frac{v_a}{3} + v_b \left[\frac{1}{3} + \frac{1}{4j} + \frac{1}{j} \right] = 0$$

$$= -\frac{v_a}{3} + v_b \left[\frac{1}{3} - \frac{j}{4} - j \right] = 0$$

$$= -\frac{v_a}{3} + v_b \left[\frac{1}{3} - \frac{5j}{4} \right] = 0$$

$$\frac{v_a}{3} = v_b \left[\frac{1}{3} - \frac{5j}{4} \right]$$

$$v_a = 3v_b \left[\frac{1}{3} - \frac{5j}{4} \right]$$

$$v_a = v_b \left[1 - \frac{15j}{4} \right] \text{ --- (3)}$$

~~$v_a =$~~

Sub eqn (2) in (1)

$$\frac{v_a}{3} - \frac{v_b}{3} = 5j$$

$$\frac{v_b}{3} (1 - \frac{15j}{4}) - \frac{v_b}{3} = -5j$$

$$v_b (1 - \frac{15j}{4}) - v_b = -5j$$

$$v_b - v_b \frac{15j}{4} - v_b = -5j$$

$$-v_b \frac{15j}{4} = -5j$$

$$v_b \frac{15j}{4} = 5$$

$$v_b = \frac{5 \times 4}{15}$$

$$v_b = \frac{20}{15}$$

$$v_b = \frac{4}{3} \text{ V}$$

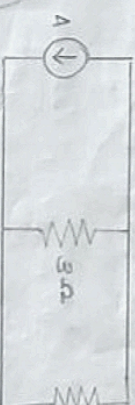
in eqn (3)

$$v_a = v_b (1 - \frac{15j}{4})$$

$$= \frac{4}{3} (1 - \frac{15j}{4})$$

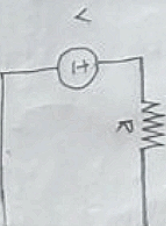
$$v_a = \frac{4}{3} - 5j$$

Qn. Determine the voltage across 10Ω resistor by applying source transformation.

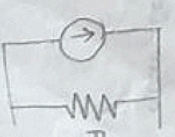


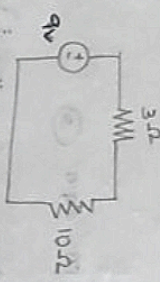
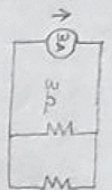
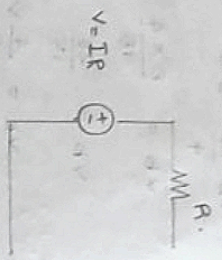
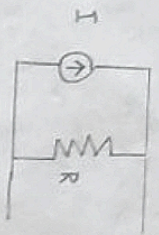
Method 1

Source transformation



$$I = V/R$$





ans:

$$V = IR = 3A \times 3\Omega = 9V$$

$$V = 9V$$

Apply voltage division rule.

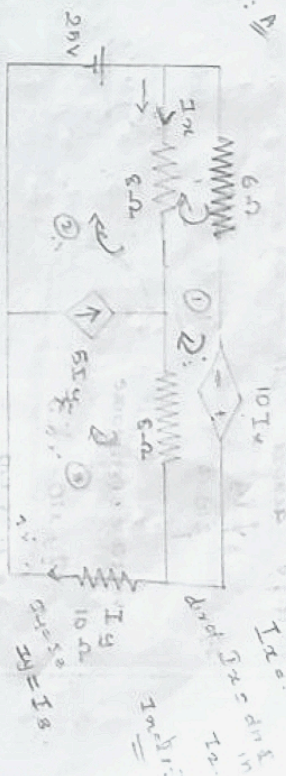
$$V_R = \frac{\text{Total voltage} \times \text{Resistance}}{\text{Total resistance}}$$

$$V_0 = \frac{9V \times 10\Omega}{13\Omega}$$

$$V_0 = \frac{9 \times 10}{13} = \frac{90}{13} V$$

Qn: Evaluate the voltage across 10Ω resistor in the following circuit.

Calculate Eqn: 1



Consider loop 1 and apply KVL

$$-6I_1 + 10I_1 - 8(I_1 - I_2) - 8(I_1 - I_2) = 0$$

$$-6I_1 + 10I_1 - 8I_1 + 8I_2 - 8I_1 + 8I_2 = 0$$

$$-22I_1 + 8I_2 + 8I_2 = -10(I_2 - I_1)$$

consider super mesh

$$I_3 - I_2 = 5I_4 \quad \text{--- (2)}$$

consider loop 2 and 3, apply KVL

$$25 - 8I_2 - 8I_1 - 8(I_2 - I_1) - 10I_2 = 0$$

$$25 - 8I_2 + 8I_1 - 8I_1 - 10I_2 = 0$$

$$16I_1 - 8I_2 - 18I_2 = -25 \quad \text{--- (3)}$$

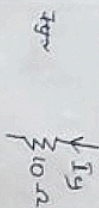
$$-22I_1 + 8I_2 + 8I_2 = -10(I_2 - I_1)$$

$$-22I_1 + 8I_2 + 8I_2 + 10I_2 - 10I_1 = 0$$

$$-32I_1 + 8I_2 + 10I_2 = 0 \quad \text{--- (1)}$$

$$I_1 = -2.71A \quad I_2 = 1.38A$$

voltage across 10Ω .



voltage resistance

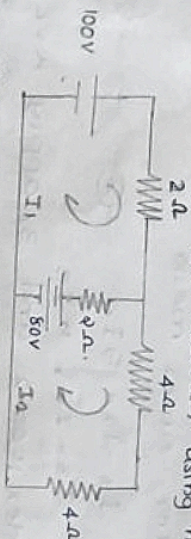
$$I_y \times 10$$

$$= 1.38 \times 10$$

$$= \underline{13.8 \text{ V.}} \quad \left. \vphantom{\begin{matrix} 1.38 \times 10 \\ 13.8 \end{matrix}} \right\} 6 \text{ Marks}$$

22/09/22
Thursday

Find current through 2Ω resistor using mesh analysis.



consider loop 1 and apply KCL

$$100 - 2I_1 - 2(I_1 - I_2) - 80 = 0$$

$$100 - 2I_1 - 2I_1 + 2I_2 = 80$$

$$100 - 4I_1 + 2I_2 = 80$$

$$-4I_1 + 2I_2 = 80 - 100$$

$$-4I_1 + 2I_2 = -20 \quad \text{--- (1)}$$

consider loop 2. KCL and apply KCL.

$$80 - 2(I_2 - I_1) - 4I_2 - 4I_2 = 0$$

$$80 - 2I_2 + 2I_1 - 4I_2 - 4I_2 = 0$$

$$80 + 2I_1 - 10I_2 = 0$$

$$2I_1 - 10I_2 = -80 \quad \text{--- (2)}$$

Solve eqn (1) and (2). we get

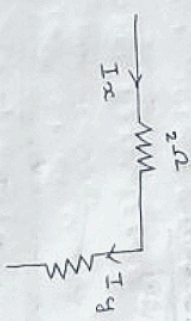
$$-4I_1 + 2I_2 = -20$$

$$2I_1 - 10I_2 = -80$$

$$I_1 = 10 \text{ A}$$

$$I_2 = 10 \text{ A}$$

current through 2Ω resistor.



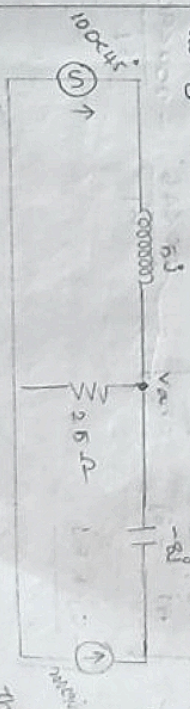
$$I_x = I_1 = 10 \text{ A}$$

$$I_y = I_2 = 10 \text{ A}$$

$$I_{ac} \cdot I_y = I_1 - I_2 = 10 - 10 = 0$$

current through 2Ω resistor = 0.

Qn Evaluate the current through 25Ω resistor using nodal analysis.



$$\frac{V_1 - 100}{2} + \frac{V_1 - V_2}{25} + \frac{V_1 - 25}{25} = 0$$

nodal analysis
does not
give
current
they always
give the
voltage

$$\frac{V_2 - 100 \angle 45^\circ}{5j} + \frac{V_2 - 0}{25} + \frac{V_1 - 200 \angle 90^\circ}{-8j}$$

$$\frac{V_1}{5j} - \frac{100 \angle 45^\circ}{5j} + \frac{V_1}{25} + \frac{V_1}{-8j} = \frac{200 \angle 90^\circ}{-8j}$$

$$\frac{V_1}{5j} - \frac{100 \angle 45^\circ}{5j} = 14.14 - 14.14j$$

$$\frac{200 \angle 90^\circ}{-8j} = \frac{200 \angle 90^\circ}{8j}$$

$$\frac{V_1}{5j} \left[\frac{1}{5j} - \frac{1}{25} + \frac{1}{-8j} \right] = \frac{100 \angle 45^\circ - 200 \angle 90^\circ}{5j}$$

$$\frac{V_1}{5j} \left[\frac{1}{5j} - \frac{1}{25} + \frac{1}{-8j} \right] = \frac{-100 \angle 45^\circ + 200 \angle 90^\circ}{5j}$$

$$\frac{V_1}{5j} \left[\frac{1}{5j} - \frac{1}{25} + \frac{1}{-8j} \right] = \frac{100 \angle 45^\circ - 200 \angle 90^\circ}{5j}$$

$$\frac{V_1}{5j} \left[\frac{1}{5j} - \frac{1}{25} + \frac{1}{-8j} \right] = \frac{100 \angle 45^\circ - 200 \angle 90^\circ}{5j}$$

$$V_1 \left[\frac{-1}{-8j + 5j} \right]$$

11/10/2020

11

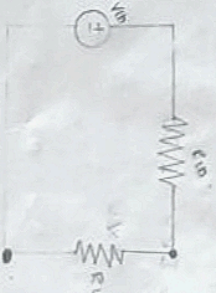
Module-2

NETWORK THEOREM:

Thevenin's theorem:

Any 2 terminal bilateral linear DC/AC circuit can be replaced by an equivalent circuit consisting of a voltage source and a series resistor.

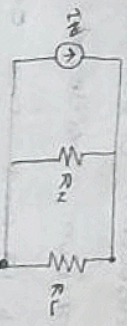
Voltage source is the open circuited voltage across the load terminals and the resistance is the internal resistance of the source network looking through open circuited load terminals.



Norton's theorem

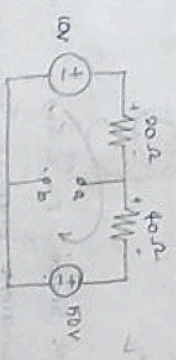
Any 2 terminal bilateral linear AC or DC network can be replaced by an equivalent circuit consisting of a current source in parallel with a resistor.

The current source is the short circuited current across the load terminal and the resistance is the internal resistance of the source network looking through the open circuited load terminal.



Go: 20/10/2020
Thursday

Find the thevenin equivalent of the given circuit w.r.t the terminals a and b



$V_{th} = V_{ab}$ = total voltage in left side elements

O_1 = total voltage in right side elements

$V_{ab} = 10V$ - Voltage across 20Ω

= - voltage across $40\Omega - 50V$

voltage across $20\Omega = I \times 20$

for finding I consider KVL

$$10 - 20I - 40I - 50 = 0$$

$$-60I - 40 = 0$$

$$-60I = 40$$

$$I = \frac{40}{-60}$$

$$-60$$

$$I = \frac{4}{-6}$$

$$-6$$

$$I = \frac{2}{-3}$$

$$\frac{2}{-3}$$

voltage across $20\Omega = \frac{2}{-3} \times 20 = \underline{\underline{13.3V}}$

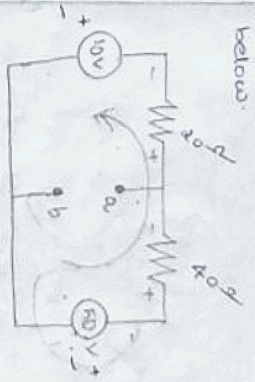
Here the direction of current is opposite

to the direction considered. When the emf

sign for V_{ab} is and the circuit is given

$$I = \frac{V}{R}$$

below



$V_{ab} = -10V$ - voltage across 20Ω

or

- voltage across $40\Omega + 50V$

$V_{ab} = 10V$ - voltage across 20Ω

= $-10 - 13.3 = \underline{\underline{-23.3V}}$

voltage across $40\Omega = \frac{2}{-3} \times 40$

$$= 50$$

$$\frac{50}{40}$$

$$= \frac{5}{4} = 1.25$$

OR

$V_{ab} = 50V$ - voltage across 40Ω

$$= 50 - 26.6V$$

$$= 23.4V$$

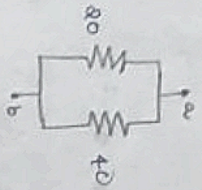
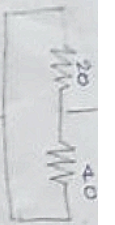
$$50 - 40 \times I$$

$$50 - 40 \times 1.25$$



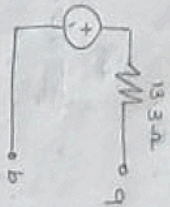
V.S Short circuit

C.S Open circuit



20 and 40 are parallel

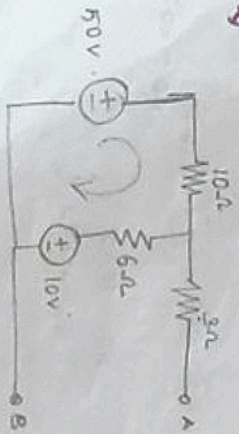
$$\frac{R_1 R_2}{R_1 + R_2} = \frac{80 \times 40}{80 + 40} = \frac{800}{120} = \frac{80}{12} = 13.3 \Omega$$



Equivalent circuit

$$\begin{aligned} \frac{V_1}{2} + \frac{V_1 - V_2}{2} &= 6 \\ \frac{V_1}{2} + \frac{V_2 - V_1}{2} &= 4 \end{aligned}$$

$$\frac{V_1 - 0}{2} + \frac{V_1 - V_2}{2} =$$



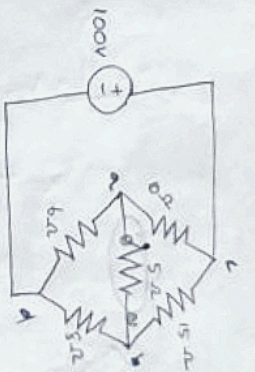
$$-2 + \frac{V_1 - 0}{2} + \frac{V_2 - 0}{2} - 4 = 0$$

$$\frac{V_1}{2} + \frac{V_2 - V_1}{2} = 6$$

$$\frac{V_1}{2} + \frac{V_2 - V_1}{2} = 6$$

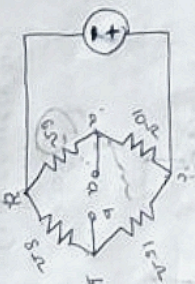
$$\frac{V_2 - V_1}{2} + \frac{V_2 - 0}{2} = -7 \Rightarrow 0$$

2

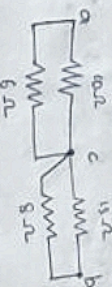


use thevenin theorem to find the current through 5 ohm resistor

E_{th}
Voltage source + short circuit
current source $\rightarrow$ Open circuit



and d are parallel



10 and 6 are parallel
15 and 8 are parallel

$$R_{th} = \frac{R_1 \times R_2}{R_1 + R_2}$$

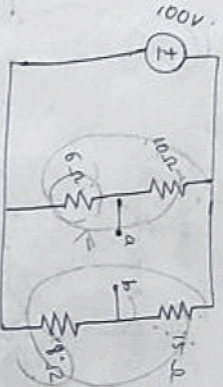
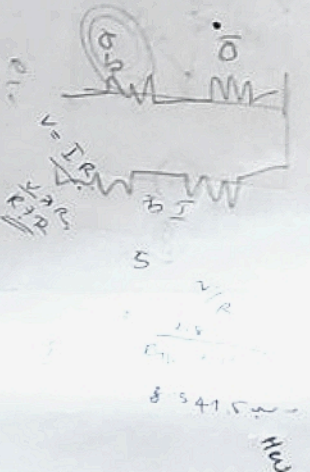
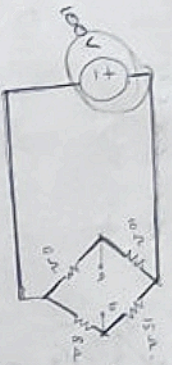
$$\frac{10 \times 6}{10 + 6} + \frac{15 \times 8}{15 + 8}$$

$$= \frac{60}{16} + \frac{120}{23}$$

$$= 8.91 \Omega$$



V4b



voltage across (10Ω resistor) = 100V

(1111 connection)

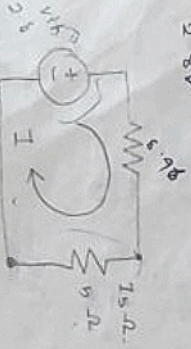
$$V_{6\Omega} = \frac{100 \times 6}{6+10} = \frac{600}{16} = 37.5V$$

But (15Ω resistor)

$$V_{8\Omega} = \frac{100 \times 8}{15+8} = \frac{800}{23} = 34.7V$$

$$V_{ab} = 6V_{6\Omega} - V_{8\Omega} = 37.5 - 34.7$$

$$V_{ab} = 2.8V$$



$$I_{6\Omega} = \frac{V_{4b}}{6\Omega} = \frac{2.8}{6} = 0.467A$$



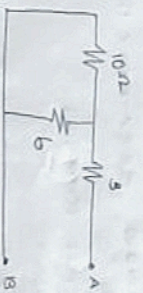
$$R_{th} = 10\Omega + 15\Omega = 25\Omega$$

$$40 - 161 = 0$$

$$I = \frac{40}{16} = 2.5A$$

$$I = 2.5A$$

Short circuit VS



10 and 6 are in series with 3

$$R_{th} = 10\Omega + 6\Omega = 16\Omega$$

$$= 16\Omega$$

$$V_{th} = V_{ab}$$

voltage across 15Ω resistor, 6Ω resistor, 10V

$$V_{ab} = 15V_{3\Omega} + 6V_{6\Omega} + 10V$$

There is no current flow through 3Ω resistor

$$V_{ab} = 0$$

$$V_{ab} = 15V_{3\Omega} + 6V_{6\Omega} + 10V$$

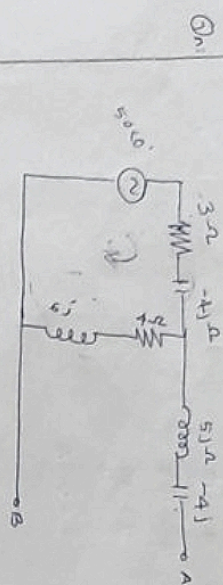
voltage across 6Ω resistor

$$V = IR$$

$$V = 2.5 \times 6$$

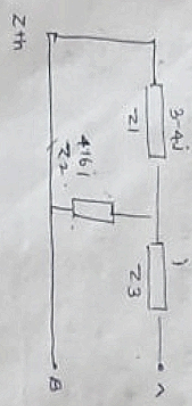
$$V = 15 \text{ V}$$

$$V_{AB} = V_{6\Omega + 10\Omega}$$



For the circuit given below determine the Thevenin equivalent b/w the output terminals.

Z_{th} voltage source circuit



$$Z_{th} = Z_{AB}$$

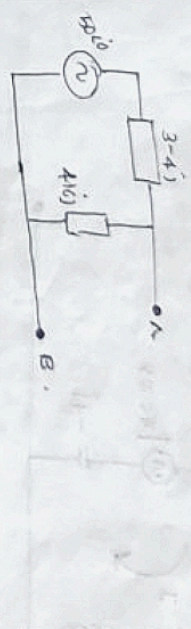
Z_1 and Z_2 in parallel series to Z_3 .

$$Z_{AB} = \frac{Z_1 Z_2}{Z_1 + Z_2} + Z_3$$

$$\frac{(3-j)(4+j)}{(3-j) + (4+j)} + j$$

$$\frac{(3-j)(4+j)}{1+j} \text{ } \Omega$$

V_{th}



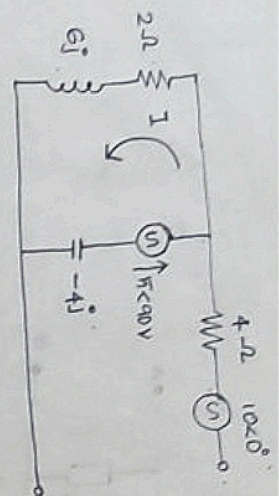
$$V_{th} = V_{AB} = V_{(4+j)\Omega}$$

$$V_{(4+j)\Omega} = \frac{50 \angle 0^\circ \times \frac{4+j}{1+j}}{4+j+3-j}$$

$$= \frac{50 \angle 0^\circ \times \frac{4+j}{1+j}}{7+j}$$

$$= \underline{\underline{(37.73 \angle 32.07^\circ) \Omega}}$$

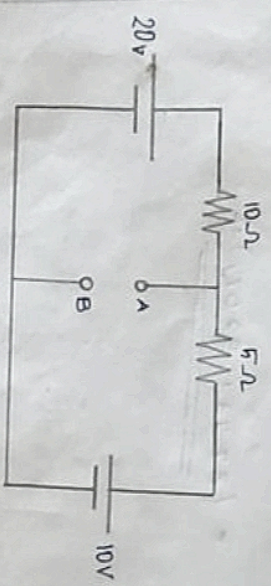
Hand *



For this circuit determine thevenin equivalent circuit.

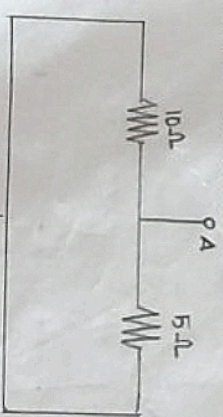
Norton's theorem

1. Determine the equivalent circuit?



$$R_N = R_{th}$$

Voltage source - short circuited



10 and 5 are parallel

$$R_N = \frac{10 \times 5}{10 + 5} = \frac{50}{15} = 3.33 \Omega$$

I_N

short circuit load terminal



$$I_N = I_1 + I_2$$

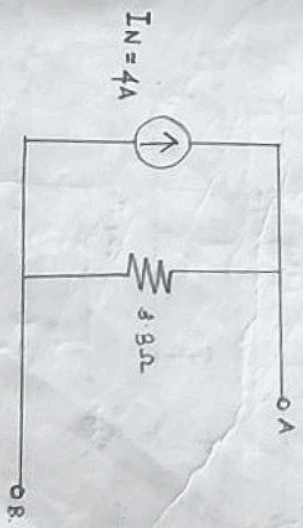
$$I_1 = \frac{V}{R} = \frac{20}{10} = 2 A$$

$$I_2 = \frac{V}{R} = \frac{10}{5} = 2 A$$

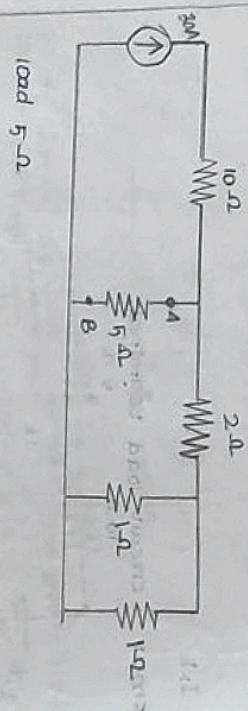
$$I_N = I_1 + I_2$$

$$I_N = 2 + 2 = 4 A$$

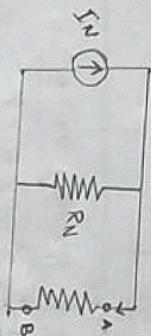
Equivalent circuit



→ Determine the current flowing through the 5Ω resistor in the given circuit by using Norton's theorem.



load 5Ω



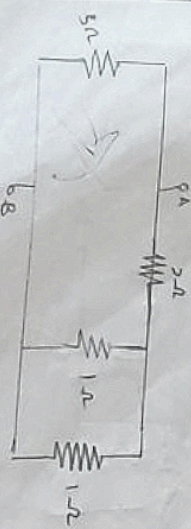
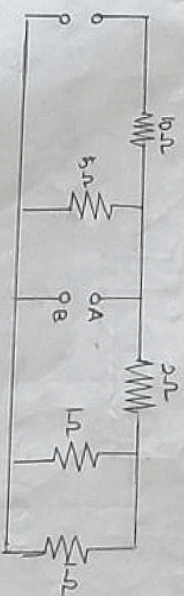
R_N

(1) load open circuit

(2) Remove source

current source - open circuit

(3) find the equivalent res. R_N



10Ω is not in parallel because of the end of the wire. 10Ω is in series with the 5Ω resistor. $10\Omega + 5\Omega = 15\Omega$ in parallel with the 2Ω resistor. $15\Omega \parallel 2\Omega = 1.66\Omega$. $R_N = 1.66\Omega$.

and 1 are parallel

$$\frac{1 \times 1 \times R_2}{1 + R_2} = \frac{1 \times 1}{1 + 1} = \frac{1}{2}$$

0.5 Ω 2 series

$$0.5 + 2 = 2.5\Omega$$

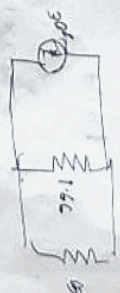
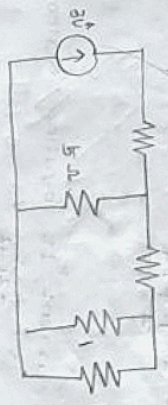
2.5Ω & 5Ω are parallel

$$R_{NB} = \frac{2.5 \times 5}{2.5 + 5} = 1.66\Omega$$

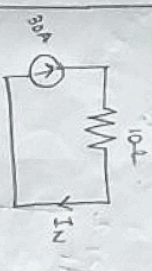
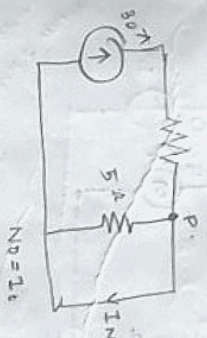
$$R_N = 1.66\Omega$$

I_N

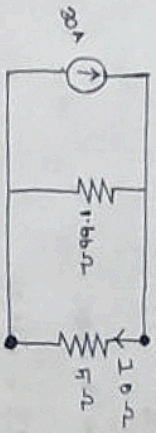
short circuit load



$$\frac{30 \times 10 \times 5}{5 + 10} = 10A$$



Determine the load current I_L by using Norton's theorem.



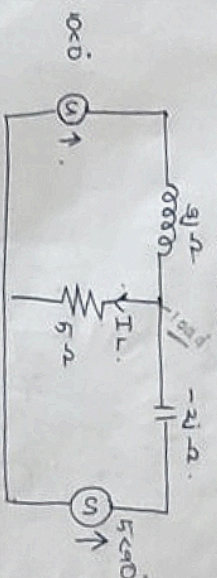
current flow through 5Ω resistor.

$$I_{5\Omega}$$

$$= \frac{30 \times 10}{10 + 10} = 1.5A$$

$$I_L = \frac{1 \times 1}{1 + 1} = 0.5A$$

→ Determine the load current I_L by using Norton's theorem.



$$I_{5\Omega}$$



source must be removed to find R_{th}

$$R_{ab} = \frac{3 \times 2}{3 + 2} = 1.2\Omega$$

$$= \frac{-6j^2}{j} = 6j$$

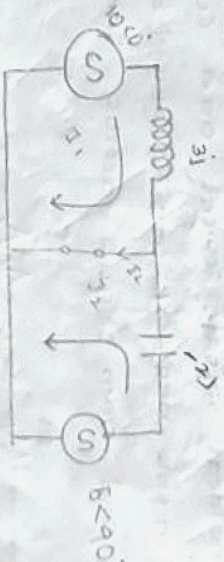
$$= \frac{-6(-1)}{j} = \frac{6}{j} = -6j$$

$$= \frac{+6}{j} = -6j$$

$$= -6j$$

for I_N

I_N



$$I = \frac{V}{R}$$

$$I_1 = \frac{10}{3} = 3.33A$$

$$I_2 = \frac{5}{2} = 2.5A$$

$$I_N = \frac{I_1 + I_2}{R_{th}}$$

$$= \frac{-10}{3}$$

$$= \frac{-5}{2}$$

$$= -3.33A$$

$$= -2.5A$$

$$= \frac{-3.33 - 2.5}{1.2} = -4.78A$$

$$I_N = I_1 + I_2$$

$$= -3.33 - 2.5 = -5.83A$$

polar form

$$3.19 \angle -166.19^\circ$$

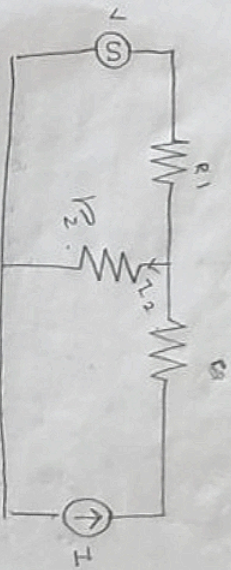
Shift + (1)

$$\text{pol}(a, b) =$$

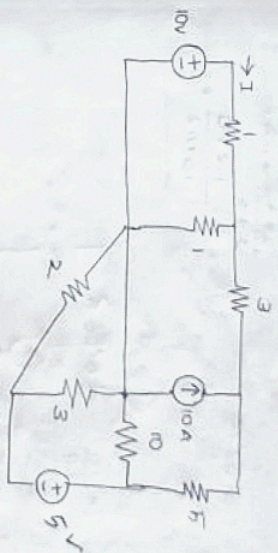
Superposition theorem

It states that in a linear network containing

2 or more sources, the response in any element is equal to the algebraic sum of the responses caused by individual sources acting alone with the other sources are non active (replaced by short circuit and open circuit across their terminals).



Determine the current I in the circuit using superposition theorem



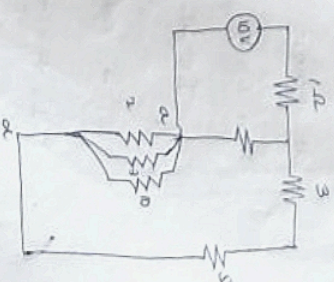
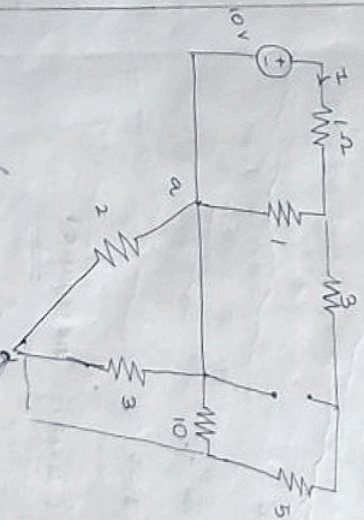
$$I = I_1 + I_2 + I_3$$

I_1 = current through 1Ω resistor when $10V$ is active and $10A$ and $5V$ is inactive

I_2 = current through 1Ω (5V is active and $10V$ & $10A$ is inactive)

I_3 = $10A$ active and $10V$ and $5V$ is inactive.

10V active



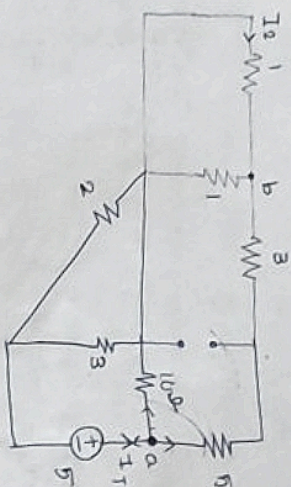
2Ω , 3Ω and 10Ω are parallel

$$\Rightarrow \frac{2 \times 3}{2+3} = 1.2 \quad 10\Omega$$

$$\Rightarrow \frac{1.2 \times 10}{1.2+10} = 1.07\Omega$$

$$I_1 = V/R_{eq} = \frac{10}{1.9} = 5.26A$$

$I_2 \Rightarrow 5V$ is active



$$I = V/R_{eq}$$

at point b' I_T divide

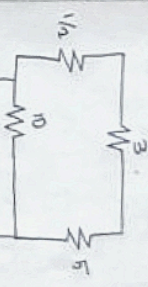
Then $I_{5\Omega}$

$$I_{5\Omega} = I_{3\Omega}$$

$I_1 \rightarrow$ current @ b'

$I_2 \rightarrow$ -ve of I_1

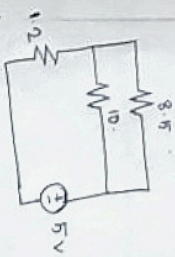
$R_{eq} = 1.912\Omega$ and (2.455) are parallel



$$\frac{1 \times 1}{1+1} = \frac{1}{2}$$

$1/2$, 3 and 5 series

$$0.5 + 3 + 5 = 8.5\Omega$$



$$1.85 \text{ (} 10 \text{)} \text{ are parallel}$$

4.6 and 1.2 series

$$R_{eq} = 4.6 + 1.2 = 5.8\Omega$$

$$I_1 = V/R_{eq} = \frac{5}{5.8} = 0.86A$$

at a apply current division

$$I_{5\Omega} = \frac{I_1 \times 10}{10+5} = \frac{0.86 \times 10}{15} = 0.57A$$

$$I_{3\Omega} = I_{5\Omega} = 0.57A$$

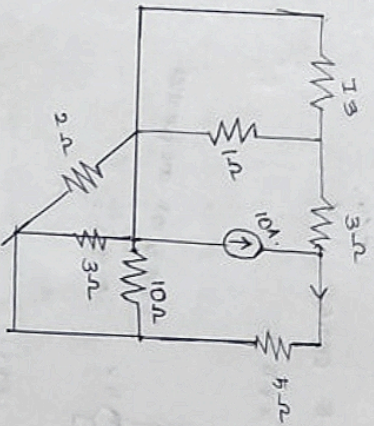
@ b' apply current division

$$I_{1\Omega} = \frac{I_{3\Omega} \times 1}{1+1} = \frac{0.57 \times 1}{2} = 0.285A$$

$$I_2 = -1.2 \text{ A} = \underline{\underline{-0.285 \text{ A}}}$$

I_3 case 3:

consider 10 A in circuit.



$$I_{3\Omega} = \frac{10 \times 5}{5+3} = \frac{50}{8} = \underline{\underline{6.25 \text{ A}}}$$

@ a current divides:

$$I_{1\Omega} = \frac{6.25 \times 1}{1+1} = \frac{6.25}{2} = \underline{\underline{3.125 \text{ A}}}$$

$$I_3 = -1.2 \text{ A} = \underline{\underline{-3.125 \text{ A}}}$$

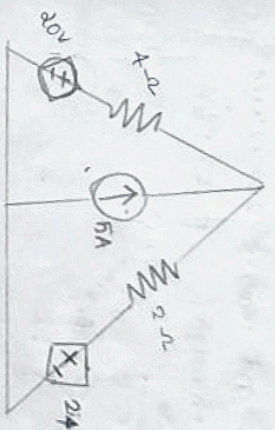
Based on super position theorem.

$$I = I_1 + I_2 + I_3$$

$$I = 5.26 + (-0.285) + (-3.125) = \underline{\underline{1.85 \text{ A}}}$$

Qn: find the value I for super position theorem

Qn:



soln: 2 Independent sources = 20V and 5A

case ①:

consider 20V and remove 5A

current flowing through $4\Omega = I_1$



$$I_4 = I_1$$

$$20 - 4I_1 - 2I_1 - 2I_1 = 0$$

$$20 - 4I_1 - 2I_1 - 2I_1 = 0 \quad I_4 = I_1$$

$$20 - 4I_1 - 2I_1 - 2I_1 = 0$$

$$20 - 8I_1 = 0$$

$$-8I_1 = -20$$

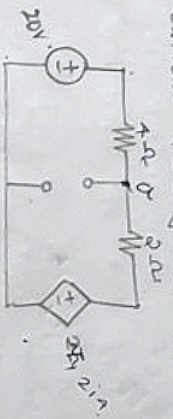
$$8I_1 = 20$$

$$I = \frac{20}{8}$$

$$I = \underline{\underline{2.5 \text{ A}}}$$

Case 2:

consider is a current and remove 20V source
current flowing through $4\Omega = I_a$



Apply KCL @ a'

$$\frac{V_a}{4} + \frac{V_a}{2} + \frac{V_a - 2I_a}{2} = 0$$

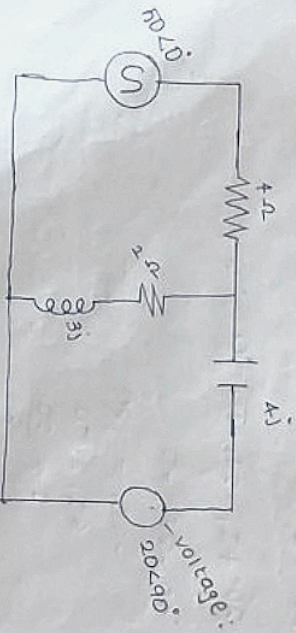
$$\frac{V_a}{4} - I_a + \frac{V_a - 2I_a}{2} = 0$$

Actual $I = I_a$ (left to right)

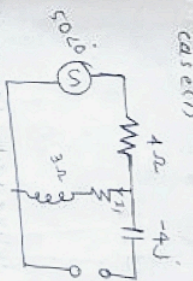
$$I_a - 0 = \frac{V_a}{4} = -\frac{V_a}{4}$$

$$\textcircled{1} \Rightarrow \frac{1}{4} V_a - I_a + \frac{1}{2} V_a + \frac{V_a}{4} = 0$$

Qn: Determine the current in at $3j\Omega$ by using Super position theorem



Case (i)



current $I_1 = \frac{V}{Z}$

$4\Omega, 3j\Omega, 2\Omega$ are series

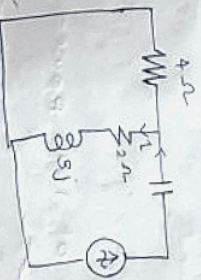
$$Z = 4 + 2 + 3j$$

$$= 6 + 3j$$

$$I_1 = \frac{50 \angle 0^\circ}{6 + 3j}$$

$$I_1 = 6.66 - 3.33j$$

Case (ii)



$$I_2 = \frac{20 \angle 90^\circ \times 4}{2 + 3j + 4}$$

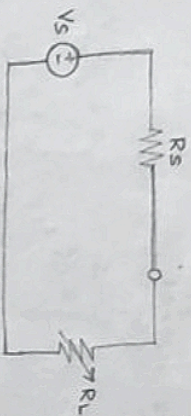
$$= 5.33 + 10.66j$$

$$I = I_1 + I_2$$

$$= 11.99 + 7.33j = 14.05 \angle 31.48^\circ$$

Maximum Power Transfer theorem

It states that maximum power is delivered from source to load when the load resistance is equal to the source resistance



Current in the circuit = I

$$I = \frac{V_s}{R_s + R_L}$$

We know that Power in load $P = I^2 R_L$

$$P_L = \frac{V_s^2}{(R_s + R_L)^2} \cdot R_L$$

For getting maximum value of load power

$$\frac{dP_L}{dR_L} = 0$$

$$\frac{d}{dR_L} \left\{ \frac{V_s^2 R_L}{(R_s + R_L)^2} \right\} = 0$$

11/10/22
Tuesday

$$\frac{D_y \times d(N_y) - N_y \times d(D_y)}{D_y^2}$$

$$D_y^2$$

Denominator x derivative (numerator) - Numerator x derivative of denominator

(Denominator)²

$$\frac{(R_s + R_L)^2 \times V_s^2 - V_s^2 R_L \times 2(R_L + R_s)}{(R_s + R_L)^4} = 0$$

$$((R_s + R_L)^2)^2$$

$$(R_s + R_L)^2 \times V_s^2 = V_s^2 R_L \times 2(R_L + R_s)$$

$$R_s^2 + R_L^2 + 2R_s R_L = 2R_L^2 + 2R_s R_L$$

$$R_s^2 + R_L^2 = 2R_L^2$$

$$R_s^2 = 2R_L^2 - R_L^2$$

$$R_s^2 = R_L^2$$

$$R_s = R_L$$



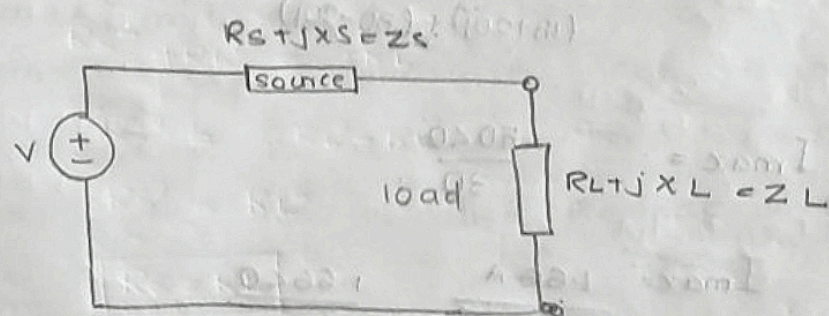
$$V = IR$$

$$I = \frac{V}{R}$$

$$P = I^2 R$$

$$P = \left(\frac{V}{R_s + R_L} \right)^2 R_L$$

Maximum power transfer theorem in AC circuit.



capacitor
+ inductor
= impedance

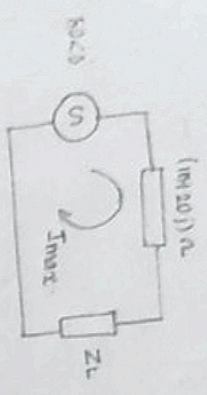
It states that maximum power is transmitted from source to load when the load impedance is equal to the conjugate of source impedance.

$$Z_L = Z_s^*$$

$$P_{max} = I^2 R_L$$

13/10/2022
Thursday

For a circuit shown below find the value of load impedance for which the source delivered the maximum power. calculate the value of maximum power.



$$Z_L = Z_S^*$$

$$Z_L = (15 + j20)^*$$

$$Z_L = 15 - j20 \Omega$$

$$P_{max} = I_{max}^2 Z_L$$

$$I = V/R$$

$$I_{max} = \frac{50}{30 + (15 - j20)}$$

$$I_{max} = \frac{50 \angle 0^\circ}{30}$$

$$I_{max} = 1.66 \text{ A}$$

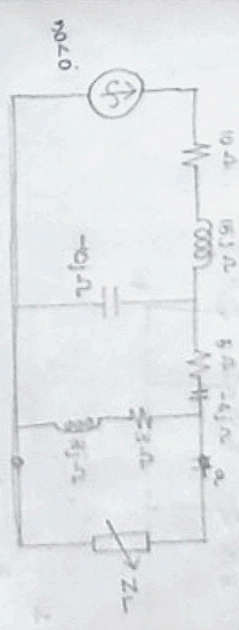
$$P_{max} = (1.66)^2 (15 - j20)$$

$$= 2.7756 (15 - j20)$$

$$= 41.33 \text{ W}$$

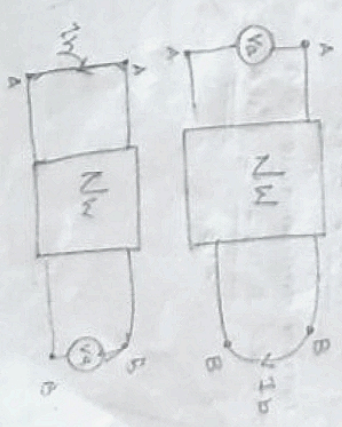
$$P_{max} = I^2 R_L$$

Qm. Determine the maximum power delivered to the given load to the given circuit.



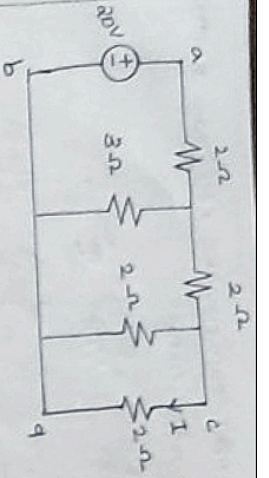
Reciprocity theorem

In any linear bilateral circuit if a single voltage source V_a in branch A produces a current I_b in branch B, then if the voltage source V_a is removed from branch A and inserted in branch B will produce a current I_b in branch A. The ratio of response to excitation is same for the 2 conditions mentioned above this is called Reciprocity theorem.

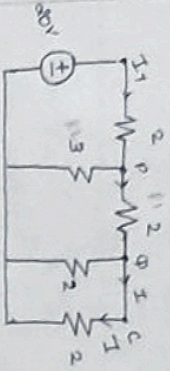


$$\frac{\text{Response}}{\text{Excitation}} = \frac{I_b}{V_a}$$

Qn:



case 1



$R_{eq} = 2\Omega \text{ and } 2\Omega \text{ are } \parallel$

$$\frac{2+2}{2 \times 2} = \frac{4}{4} = 1$$

1 and 2 are in series
current divided

$$1+2=3$$

rule

$$\frac{3 \times 3}{3+3} = \frac{9}{6} = 1.5 \quad \text{total current}$$

$$1.5+2=3.5$$

$$1.5+2=3.5$$

$$I_1 = \frac{V}{R_{eq}}$$

$$V = 20V$$

$$I_1 = \frac{20}{3.5} = 5.71$$

$$I_1 = 5.71A$$

case: current division rule @ P

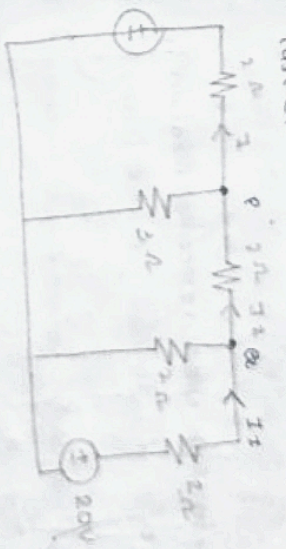
$$I_{2\Omega} = \frac{5.71 \times 3}{3+2}$$

$$= \frac{17.13}{5} = 3.42A$$

current division rule @ Q

$$I_{2\Omega} = \frac{5.71 \times 2}{2+3} = 1.71A$$

case 2:



$$I_1 = \frac{V}{R_{eq}}$$

$$V = 20$$

$$R_{eq} = [2 \parallel 2 + 2]$$

$$R_{eq} = \{ [3 \parallel 2 + 2] \parallel 2 \} + 2$$

$$\left\{ \left[\frac{3 \times 2}{3+2} + 2 \right] \parallel 2 \right\} + 2$$

$$\left\{ \left[\frac{6}{5} + 2 \right] \parallel 2 \right\} + 2$$

$$= 3.2\Omega$$

I = current

$$I_1 = \frac{20}{3.2} = 6.25A$$

$I_{2\Omega}$ = current division @ Q

$$= \frac{I_1 \times 2}{2+2} = \frac{6.25 \times 2}{4} = 3.12A$$

$I_{2\Omega}$ = current division @ P

$$\frac{I_1 \times 3}{3+2} = \frac{6.25 \times 3}{5} = 1.87A$$

For both cases the ratio of response to excitation is $\frac{1.6}{20}$.

Hence we can say the reciprocity theorem is valid in this circuit.

Module-7

LAPLACE TRANSFORMATION.

The time function is usually designated by lower case letters and laplace transformation by capital letters. It represents the complex frequency (s) domain. analysis i.e. $s = \sigma + j\omega$.

Here we consider $f(t)$ is a function in time domain we can convert this time domain function to frequency domain function $F(s)$ by using laplace transformation.

$$F(s) = \int_0^{\infty} f(t) e^{-st} dt$$

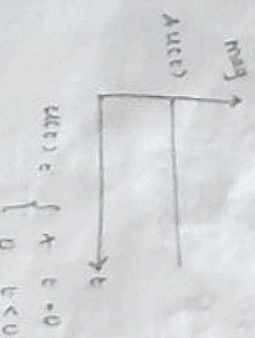
Inverse laplace transformation.

It permits the reverse operation that means frequency domain to the domain analysis.

$$L^{-1}[F(s)] = f(t)$$

Common forcing functions.

1. Step signal $u(t)$.

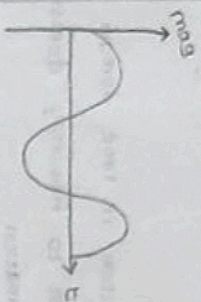


Unit Step signal (u(t))

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

2. Exponential function

3. Sinusoidal function



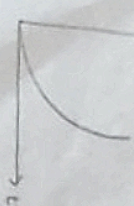
$$f(t) = \sin \omega t$$

4. Ramp function



$$r(t) = \begin{cases} t, & t > 0 \\ 0, & t \leq 0 \end{cases}$$

A. Parabolic function



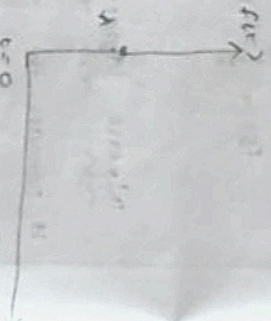
$$f(t) = \begin{cases} t^2, & t > 0 \\ 0, & t \leq 0 \end{cases}$$

B.



5t

6. Impulse signal



$$f(t) = \begin{cases} A, & t = 0 \\ 0, & t \neq 0 \end{cases}$$

$f(t) = u(t) - u(t-T)$

Function in time domain	LT (s domain)
$u(t)$	$\frac{1}{s}$
$\delta(t)$	1
t^n	$\frac{n!}{s^{n+1}}$
t	$\frac{1}{s^2}$
e^{at}	$\frac{1}{s-a}$
e^{at}	$\frac{1}{s-a}$
$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
$e^{at} \cos \omega t$	$\frac{(s-a)}{(s^2 + \omega^2)}$
Step signal $u(t)$	$\frac{1}{s}$
Impulse signal $\delta(t)$	1
$L[\delta(t)]$	1
$L[\frac{d}{dt} f(t)]$	$s f(s) - f(0)$
$L[\int_0^t f(\tau) d\tau]$	$\frac{1}{s} f(s) + \frac{f(0)}{s}$

$u(t)$

$\frac{1}{s}$

$\frac{1}{s} f(s) + \frac{f(0)}{s}$

Final value theorem

Initial value theorem

It gives the initial value of the system in a time domain

$$\lim_{t \rightarrow 0} f(t) = \lim_{s \rightarrow \infty} s f(s)$$

In laplace form the initial value is

$$\lim_{s \rightarrow \infty} s f(s)$$

$$\lim_{t \rightarrow 0} f(t) = \lim_{s \rightarrow \infty} s f(s)$$

Final value theorem

It gives the final value of the system in time domain the final value give the expression

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} s f(s)$$

In frequency domain the final value is

$$\lim_{s \rightarrow 0} s f(s)$$

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} s f(s)$$

Initial value theorem

1. Obtain the laplace transform of $f(t) = 1 - e^{-at}$

ans:

$$L(f(t)) = L[1 - e^{-at}] = L[1] - L[e^{-at}]$$

$$= \frac{1}{s} - \frac{1}{s+a}$$

$$= \frac{s+a-s}{s(s+a)}$$

$$= \frac{a}{s(s+a)}$$

find the inverse

2. Laplace transform of $\frac{2}{s} - \frac{1}{s+3}$

ans:

$$L^{-1}(f(s)) = L^{-1}\left[\frac{2}{s} - \frac{1}{s+3}\right]$$

$$= L^{-1}\left[\frac{2}{s} - \frac{1}{s+3}\right]$$

$$= 2 - e^{-3t}$$

important:

3.

A shifted unit step function is $f(t) = u(t-1)$ Obtain its laplace transform

$$L(u(t)) = \frac{1}{s}$$

$$L(u(t-1)) = \frac{1}{s} e^{-s}$$

$$L(u(t-1)) = \frac{1}{s} e^{-s}$$

Shifting function $\Rightarrow e^{-s}$

function

$$u(t-1)$$

$$L(u(t)) = L(t)$$

amp

$$= \frac{1}{s^2}$$

$$\left\{ L(u(t)) \right\} = \frac{1}{s^2} \times e^{-st}$$

$$L(s(t)) = 1$$

$$L(s(t-1)) = e^{-st}$$

u(t)

4:

A half cycle sine wave function is given by.

*

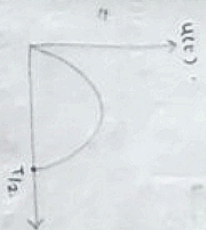
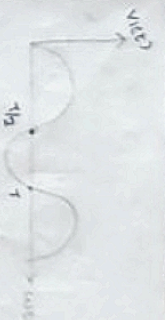
$V(t) = \sin \omega t$ determine its Laplace transform.



SCM



for the removal of -ve part



$$V_1(t) + V_2(t) = V(t)$$

$$V_1(t) = \sin \omega t \quad u(t)$$

$$\omega = 2\pi f$$

$$f = \frac{1}{T}$$

$$\omega = \frac{2\pi}{T}$$

$$V_1(t) = \sin \omega t \quad u(t)$$

$$V_2(t) = \sin \left(\frac{2\pi}{T} t - \frac{\pi}{2} \right) u(t - T/2)$$

adding laplace transform on both side.

$$\sin \frac{2\pi}{T} t$$

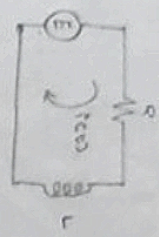
$$\sin \frac{2\pi}{T} t = \frac{2\pi}{T} \times \frac{1}{s^2}$$

$$\left[\sin \left(\frac{2\pi}{T} t - \frac{\pi}{2} \right) u(t - T/2) \right] + \left[\sin \frac{2\pi}{T} t \quad u(t) \right]$$

$$V(t) = \left[\frac{2\pi}{T} \times \frac{1}{s^2} \right] \times \left[\frac{1}{s^2} \times \frac{1}{s} \right]$$

17/10/22
Monday

Step response of RL series circuit



$E \rightarrow$ step, $L \rightarrow$ de energised form

Apply KVL

$$E u(t) - R i(t) - L \frac{di(t)}{dt} = 0$$

$$E u(t) = R i(t) + L \frac{di(t)}{dt}$$

Taking Laplace transform

$$\frac{E}{s} = R I(s) + L \{ s I(s) - i(0) \}$$

Initial condition $i(0) = 0$

$$\frac{E}{s} = R I(s) + L s I(s)$$

$$\frac{E}{s} = (R + Ls) I(s)$$

$$I(s) = \frac{E}{s(R + Ls)}$$

Use

$$\frac{E}{s(R + Ls)} \times L$$

$$= \frac{E}{Ls(R + Ls)}$$

$$I(s) = \frac{E}{L} \left\{ \frac{1}{s(R + Ls)} \right\}$$

Taking inverse Laplace transform

Answer

Taking partial fraction of

$$\frac{1}{s(R + Ls)} = \frac{A}{s} + \frac{B}{s + R/L}$$

$$\frac{s(R + Ls)}{s(R + Ls)} = \frac{A s(R + Ls)}{s(R + Ls)} + \frac{B s(R + Ls)}{(s + R/L)(R + Ls)}$$

$$1 = A(s + R/L) + B s$$

$$1 = A s + A R/L + B s$$

Equating the coefficients on LHS and RHS

$$1 = A R/L$$

$$A = \frac{L}{R}$$

Equating the coefficient of s

$$A + B = 0$$

$$B = -A$$

$$B = -\frac{L}{R}$$

$$\frac{1}{s(R + Ls)} = \frac{\frac{L}{R}}{s} + \frac{-\frac{L}{R}}{s + R/L}$$

$$\frac{1}{s} = u(t)$$

$$\frac{1}{s-a} = e^{at}$$

$$\frac{1}{s+a} = e^{-at}$$

$$I(t) = i(t)$$

A term which disappears on taking Laplace transform

Then

$$I_{cc}(s) = \frac{E}{L} \left\{ \frac{L/R}{s} - \frac{L/R}{s+R/L} \right\}$$

$$\frac{E L/R}{L} \left\{ \frac{1}{s} - \frac{1}{s+R/L} \right\}$$

$$I(s) = \frac{E}{R} \left\{ \frac{1}{s} - \frac{1}{s+R/L} \right\}$$

Taking inverse Laplace transform on both side

$$\rightarrow L I(s) = i(t)$$

$$\rightarrow L \left[\frac{1}{s} \right] = u(t) \text{ or } 1$$

$$\rightarrow L \left[\frac{1}{s+R/L} \right] = e^{-R/L t}$$

$$i(t) = \frac{E}{R} \left\{ u(t) - e^{-R/L t} \right\}$$

$$i(t) = \frac{E}{R} \left\{ 1 - e^{-R/L t} \right\}$$

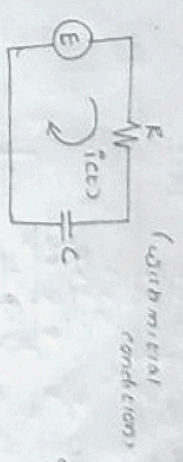
Note:

$$\frac{1}{s(s+a)} = \frac{A}{s} + \frac{B}{(s+a)}$$

$$\frac{1}{s(s+a)^2} = \frac{A}{s} + \frac{B}{(s+a)} + \frac{C}{(s+a)^2}$$

$$\frac{1}{s(s^2+4as+a^2)} = \frac{A}{s} + \frac{Bs+C}{(s^2+4as+a^2)}$$

Set response of RC series circuit:



cap voltage

$$\frac{1}{C} \int i(t) dt$$

 capacitor voltage

$$\frac{1}{C} \int i(t) dt$$

Let us assume that the capacitance of the circuit is initially having a charge Q_0 and it causes an initial current in the circuit.

Apply KVL

$$E - R i(t) - \frac{1}{C} \int i(t) dt = 0$$

$$E - R i(t) + \frac{1}{C} \int i(t) dt$$

Taking Laplace transform on both side

$$E - R i(s) = \frac{1}{C} \left[\frac{1}{s} \left(\int_0^\infty i(t) dt \right) + \frac{Q_0}{s} \right]$$

$$R i(s) = R I(s)$$

$$\frac{1}{C} \int i(t) dt = \frac{1}{C} \left[\frac{1}{s} \left(\int_0^\infty i(t) dt \right) + \frac{Q_0}{s} \right]$$

$$\frac{E}{s} = R I(s) + \frac{1}{C} \left[\frac{1}{s} \left(\int_0^\infty i(t) dt \right) + \frac{Q_0}{s} \right]$$

Initial condition $I(0) = \frac{Q_0}{C}$

$$\frac{1}{C} \int_0^\infty i(t) dt = \frac{1}{C} \int_0^\infty i(t) dt$$

$$\frac{E}{s} = R I(s) + \frac{1}{C} \left[\frac{1}{s} \left(\int_0^\infty i(t) dt \right) + \frac{Q_0}{s} \right]$$

Taking

$$\frac{E}{s} - \frac{Q_0}{Cs} = \left[R + \frac{1}{Cs} \right] I(s)$$

$$I(s) = \frac{\frac{E}{s} - \frac{Q_0}{Cs}}{R + \frac{1}{Cs}}$$

$$I(s) = \frac{\frac{1}{s} \left[E - \frac{Q_0}{C} \right]}{\frac{Rcs+1}{Cs}}$$

$$= \frac{1}{s} \left(E - \frac{Q_0}{C} \right) \times \frac{Cs}{Rcs+1}$$

$$= \frac{E - \frac{Q_0}{C}}{Rcs+1} = \frac{E - \frac{Q_0}{C}}{Rc} \frac{Rc}{Rcs+1}$$

$$= \frac{E - \frac{Q_0}{C}}{Rc(s + 1/Rc)}$$

$$I(s) = \frac{\frac{E}{R} - \frac{Q_0}{Rc}}{s + \frac{1}{Rc}}$$

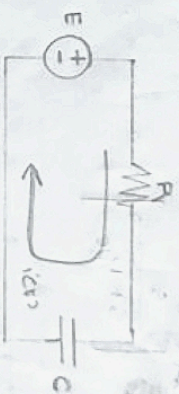
Take Initial laplace transform.

$$i(t) = \left\{ \frac{E}{R} - \frac{Q_0}{Rc} \right\} e^{-t/Rc}$$

Q1. find laplace transform of $f(t)$.

Q2. →

Impulse response of series RC Network.



soln:

$$E = f(t)$$

Applying KVL in this loop.

$$E = f(t) - Ri(t) - \frac{1}{C} \int i(t) dt = 0$$

$$E f(t) = Ri(t) + \frac{1}{C} \int i(t) dt = 0$$

Applying laplace transform

$$E = Ri(s) + \frac{1}{C} \left[\frac{I(s)}{s} + \frac{i(0)}{s} \right]$$

$$I(s) = 0$$

$$\frac{E}{s} = \left[Ri(s) + \frac{1}{C} \right] I(s)$$

$$E = \left[R + \frac{1}{Cs} \right] I(s)$$

$$I(s) = \frac{E}{\left(R + \frac{1}{Cs} \right)}$$

$$I(s) = \frac{E}{R + \frac{1}{Cs}} = \frac{Ecs}{Rcs+1}$$

$$I(s) = \frac{Ecs}{(Rcs+1) Rc}$$

$$\mathcal{L}^{-1}[\cdot] = \delta(t)$$

$$I(s) =$$

$$\frac{E/s}{R + \frac{1}{sC}}$$

$$= \frac{E}{R} \left[\frac{s}{s + \frac{1}{RC}} \right]$$

$$s + \frac{1}{RC} \left[\frac{1}{s + \frac{1}{RC}} \right]$$

$$\frac{1}{2} \quad \frac{1}{2} \quad \mathcal{L}^{-1}[\cdot]$$

$$\frac{s}{s + \frac{1}{RC}} = 1 + \frac{-\frac{1}{RC}}{s + \frac{1}{RC}}$$

$$I(s) = \frac{E}{R} \left\{ 1 - \frac{\frac{1}{RC}}{s + \frac{1}{RC}} \right\}$$

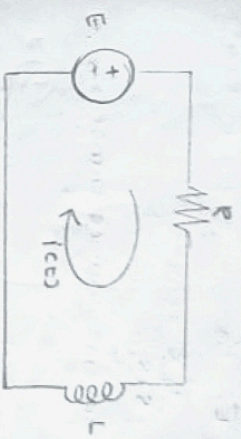
Taking I.L.T.

$$i(t) = \frac{E}{R} \left\{ \delta(t) - \frac{1}{RC} e^{-\frac{t}{RC}} \right\}$$

$$E/R \left[\delta(t) - \frac{1}{RC} e^{-\frac{t}{RC}} \right]$$

Q2 →

Impulse response of series RL network



E → step $\Rightarrow$ de energised form

Applying KVL on the loop.

$$E - Ri(t) - L \frac{di(t)}{dt} = 0$$

$$E - Ri(t) + L \frac{di(t)}{dt} = 0$$

$$\mathcal{L}[f(t)] = \frac{1}{s}$$

Taking Laplace transform.

$$\frac{E}{s} = R I(s) + L \{ s I(s) - i(0) \}$$

$\frac{E}{s}$ initial condition $i(0) = 0$

$$\frac{E}{s} = R I(s) + \{ s I(s) \}$$

$$\frac{E}{s} = (R + sL) I(s)$$

$$I(s) = \frac{E}{s(R + sL)}$$

$$I(s) = \frac{E}{s(R + sL)}$$

$$= \frac{E}{L} \frac{1}{s(R/L + s)}$$

$$I(s) = \frac{E}{L} \frac{1}{s(R/L + s)}$$

$$I(s) = \frac{E}{L} \left(\frac{1}{s + \frac{R}{L}} \right)$$

$$I(s) = \frac{E}{L} \cdot \frac{1}{s + \frac{R}{L}}$$

Taking inverse Laplace transform

$$I(t) =$$

$$I(s) = \frac{E}{L} \cdot \frac{1}{s + \frac{R}{L}}$$

$$L^{-1} I(s) = L^{-1} \left[\left(\frac{E}{L} \right) \cdot \frac{1}{s + \frac{R}{L}} \right]$$

$$i(t) = \frac{E}{L} e^{-\frac{R}{L}t} \left(\frac{1}{s + \frac{R}{L}} \right)$$

$$i(t) = \frac{E}{L} e^{-\frac{R}{L}t}$$

sinusoidal response of RL circuit



Applying KVL on the loop

$$E_m \cos(\omega t + \phi) - R i - L \frac{di}{dt} = 0$$

$$E_m \cos(\omega t + \phi) = R i + L \frac{di}{dt}$$

$$\frac{di}{dt} = D$$

$$E_m \cos(\omega t + \phi) = R i + L \frac{di}{dt}$$

$$E_m \cos(\omega t + \phi) = R i + L \frac{di}{dt}$$

$$E_m \cos(\omega t + \phi) = R i + L \frac{di}{dt} \quad \text{--- (1)}$$

$$\frac{E_m}{L} \cos(\omega t + \phi) = \frac{R}{L} i + \frac{di}{dt}$$

$$\frac{E_m}{L} \cos(\omega t + \phi) = \frac{R}{L} i + \frac{di}{dt}$$

$$\frac{E_m}{L} \cos(\omega t + \phi) = \frac{R}{L} i + \frac{di}{dt}$$

$$i \left[\frac{R}{L} + j\omega \right] = \frac{E_m}{L} \cos(\omega t + \phi)$$

its solutions has 2 terms

1) CF (complementary function)

2) PI (particular integral)

$$i = i_{CF} + i_{PI}$$

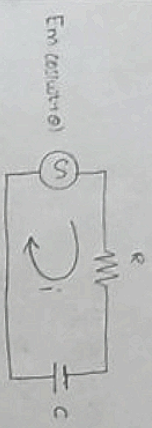
$$i_{CF} = C e^{-\frac{R}{L}t}$$

$$i_{PI} = \frac{E_m}{\sqrt{R^2 + (\omega L)^2}} \cos(\omega t + \phi - \tan^{-1} \frac{\omega L}{R})$$

$$i = i_{CF} + i_{PI}$$

$$i = \frac{E_m}{R} e^{-R/Lt} + \frac{E_m}{\sqrt{R^2 + (\omega L)^2}} [\cos(\omega t + \theta) - e^{-R/Lt} \cos \theta]$$

✓ sinusoidal response of RC circuit



Integrate and remove the terms of the integral

Apply the KVL on the loop.

$$E_m \cos(\omega t + \theta) - Ri - \frac{1}{C} \int i dt = 0$$

$$E_m \cos(\omega t + \theta) = Ri + \frac{1}{C} \int i dt$$

For the removal of integration by multiplying with differential function

$$E_m \frac{d}{dt} \cos(\omega t + \theta) = R \frac{d}{dt} i + \frac{1}{C} \cdot \frac{d}{dt} \int i dt$$

$$-E_m \sin(\omega t + \theta) \omega = R \frac{di}{dt} + \frac{1}{C} i$$

$$-E_m \omega \sin(\omega t + \theta) = R \frac{di}{dt} + \frac{1}{C} i \quad \text{--- (1)}$$

① $\div R$

$$-\frac{E_m \omega}{R} \sin(\omega t + \theta) = \frac{di}{dt} + \frac{1}{RC} i$$

$$\frac{di}{dt} = D$$

$$-\frac{E_m \omega}{R} \sin(\omega t + \theta) = D i + \frac{1}{RC} i$$

$$Di + \frac{1}{RC} i = -\frac{E_m \omega}{R} \sin(\omega t + \theta)$$

$$i \left[D + \frac{1}{RC} \right] = -\frac{E_m \omega}{R} \sin(\omega t + \theta)$$

Solution have 2 terms.

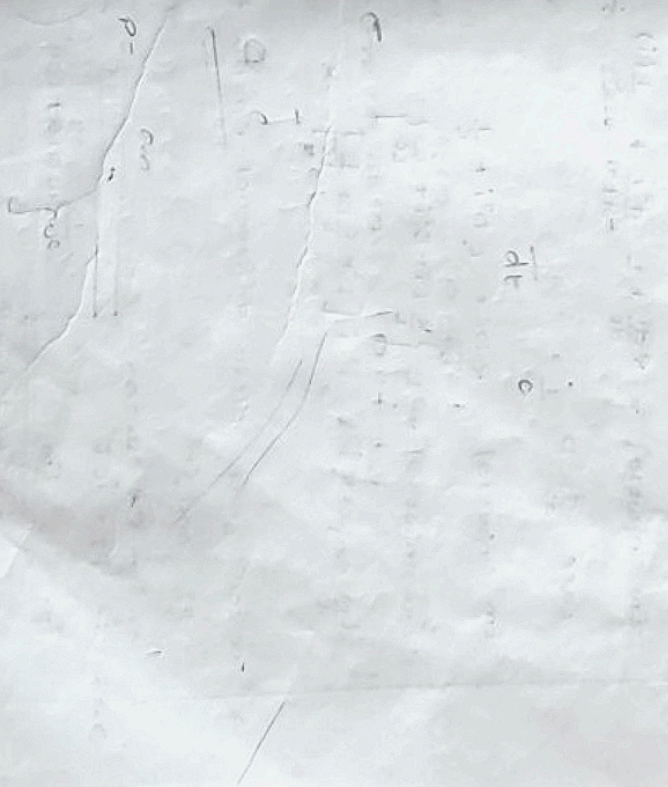
1) PI (particular integral)

2) CF (complementary function)

$$i_{CF} = C e^{-R/Lt}$$

$$i_{PI} = \frac{E_m}{\sqrt{\omega^2 R^2 + \left(\frac{R}{L}\right)^2}} \cos[\omega t + \theta + \tan^{-1} \frac{\omega L}{R}]$$

$$i = C e^{-R/Lt} + \frac{E_m}{\sqrt{R^2 + (\omega L)^2}} [\cos(\omega t + \theta + \tan^{-1} \frac{\omega L}{R})]$$



Sinusoidal response of RLC series circuit.



KVL

$$E_m \cos(\omega t + \theta) - Ri - L \frac{di}{dt} - \frac{1}{C} \int i dt = 0$$

$$E_m \cos(\omega t + \theta) = Ri + L \frac{di}{dt} + \frac{1}{C} \int i dt = 0 \cdot \frac{d}{dt}$$

$$E_m \frac{d}{dt} \cos(\omega t + \theta) = R \frac{di}{dt} + L \frac{d^2 i}{dt^2} + \frac{1}{C} \int i dt \cdot \frac{d}{dt}$$

$$-E_m \sin(\omega t + \theta) = R \frac{di}{dt} + L \frac{d^2 i}{dt^2} + \frac{1}{C} (i) \cdot iR$$

case $\frac{d}{dt} = D$

$$-E_m \sin(\omega t + \theta) = R Di + L D^2 i + \frac{1}{C} i$$

$$-E_m \sin(\omega t + \theta) = L \left\{ \frac{R}{L} Di + D^2 i + \frac{1}{LC} i \right\}$$

$$-\frac{E_m}{L} \sin(\omega t + \theta) = i \left[D^2 + \frac{R}{L} D + \frac{1}{LC} \right]$$

→ The soln of the above differential eqn has 2 parts

P1 and CF

$$i = i_{P1} + i_{CF}$$

$$i_{P1} = \frac{E_m}{\sqrt{R^2 + \left(\frac{1}{C\omega} + L\omega\right)^2}}$$

$$\cos\{\omega t + \theta + \tan^{-1}\left(\frac{\frac{1}{C\omega} - L\omega}{R}\right)\}$$

i_{CF}

Here the term

$D^2 + \frac{R}{L} D + \frac{1}{LC}$ have two roots

consider it as D_1 and D_2

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4 \times 1 \times \frac{1}{LC}}}{2 \times 1}$$

$$= \frac{-\frac{R}{L} \pm \frac{1}{2} \sqrt{\frac{R^2}{L^2} - \frac{4}{LC}}}{K_1 \quad K_2}$$

$$D_1 = K_1 + K_2$$

$$D_2 = K_1 - K_2$$

case 1:

$$\text{when } \left[\frac{R}{2L}\right]^2 > \frac{1}{LC}$$

→ The roots are real and unequal which gives Over damped response

then the complete function (K1 + K2)e^{(D1 + D2)t} + C1e^{(D1)t} + C2e^{(D2)t}

Then the total power

$$i = i_{P1} + i_{CF}$$

Case 2:

$$\text{when } \left[\frac{R}{2L}\right]^2 < \frac{1}{LC}$$

Here the roots are complex conjugate which gives under damped response.

$$\text{Here } i_C = e^{K_1 t} [C_1 \cos K_2 t + C_2 \sin K_2 t]$$

Case 3:

$$\text{when } \left[\frac{R}{2L}\right]^2 = \frac{1}{LC}$$

Here the roots are equal which gives critically damped response.

$$\text{Here } i_C = e^{K_1 t} [C_1 + C_2 t]$$

and the value of:

$$i_C = i_{p1} + i_{p2}$$

under

$$i_C = e^{K_1 t} (C_1 \cos K_2 t + C_2 \sin K_2 t)$$

$$\left[\frac{R}{2L}\right]^2 < \frac{1}{LC}$$

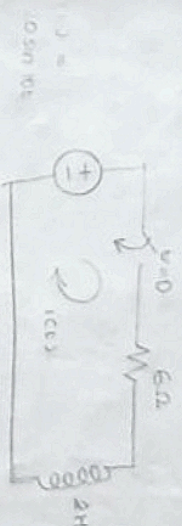
$$e^{K_1 t} (C_1 \cos K_2 t + C_2 \sin K_2 t)$$

$$i_C = \text{this}$$

$$K = \text{undamped}$$

2

Evaluate $i_C(t)$ in the network for $V_{in}(t) = 20 \sin 10t$ switch is closed at $t=0$. Assume that initial value of current through the inductor is zero.



Ans:

Apply KVL on the loop.

$$V_{in}(t) - 6i_C(t) - L \frac{di_C(t)}{dt} = 0$$

$$L = 2$$

$$V_{in}(t) = 20 \sin 10t$$

$$20 \sin 10t - 6i_C(t) - 2 \frac{di_C(t)}{dt} = 0$$

$$20 \sin 10t = 6i_C(t) + 2 \frac{di_C(t)}{dt}$$

$$20 \sin 10t = 6i_C(t) + 2 \frac{di_C(t)}{dt}$$

$$\frac{20 \times 10}{s^2 + 10^2} = 6i_C(s) + 2[s i_C(s) - i_C(0)]$$

$$\sin 10t = \frac{10}{s^2 + 10^2}$$

$$\frac{200}{s^2 + 100} = 6i_C(s) + 2[s i_C(s)]$$

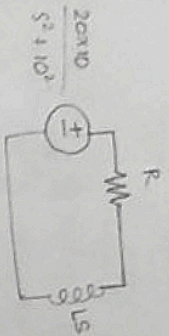
$$\frac{200}{s^2 + 100} = i_C(s)[6 + 2s]$$

$$i_C(s) = \frac{200}{[s^2 + 100][6 + 2s]}$$

$$i_C(s) = \frac{200}{2(s+3)(s^2+100)}$$

$$i_C(s) = \frac{100}{(s+3)(s^2+100)}$$

$$I(s) = \frac{100}{(s+3)(s^2+100)}$$



$$I(s) = \frac{100}{(s+3)(s^2+100)}$$

ie taking laplace transform

$$\frac{100}{(s+3)(s^2+100)} = \frac{A}{s+3} + \frac{Bs+C}{s^2+100}$$

$$\frac{100(s+3)(s^2+100)}{(s+3)(s^2+100)} = \frac{A(s+3)(s^2+100)}{(s+3)(s^2+100)} + \frac{Bs+C(s+3)(s^2+100)}{(s+3)(s^2+100)}$$

$$100 = A(s^2+100) + (Bs+C)(s+3)$$

$$100 = A(s^2+100) + (Bs+C)(s+3)$$

Equating

$$100 = As^2 + 100A + Bs^2 + 3Bs + Cs + 3C$$

Equating the constants

$$100 = 100A + 3C$$

Equating coefficient of s

$$0 = 3B + C$$

$$3B = -C$$

$$3B = -C$$

$$3B = -C$$

$$3B = -C$$

Equating the coefficient of s^2

$$0 = A + B$$

$$A + 100 + 3C = 100 \quad \text{--- (1)}$$

$$3B + C = 0 \quad \text{--- (2)}$$

$$A + B = 0 \quad \text{--- (3)}$$

$$A = 0.917 \rightarrow 0.92$$

$$B = -0.917 \rightarrow -0.92$$

$$C = 2.75$$

$$100 =$$

$$\frac{100}{(s+3)(s^2+100)} = \frac{A}{s+3} + \frac{Bs+C}{s^2+100}$$

$$\frac{100}{(s+3)(s^2+100)} = \frac{0.92}{s+3} + \frac{(-0.92)s + 2.75}{s^2+100}$$

$$= \frac{0.92}{s+3} - \frac{(0.92)s}{s^2+100} + \frac{2.75}{s^2+100}$$

$$i(s)$$

$$= 0.92e^{-3t} - (0.92) \cos 10t + (2.75) \sin 10t$$

Taking inverse transform

$$i(t) = 0.92e^{-3t} - (0.92) \cos 10t + (2.75) \sin 10t$$

1:

Verify initial and final value theorems of Laplace transform for the following function.

$$f(t) = e^{-t} \left\{ t^2 + t^3 + \sin 2t \right\}$$

Initial value theorem

$$\lim_{t \rightarrow 0} f(t) = \lim_{s \rightarrow \infty} s f(s)$$

Final value theorem

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} s f(s)$$

L.V.T

$$\lim_{t \rightarrow 0} f(t) = \lim_{t \rightarrow 0} e^{-t} \left\{ t^2 + t^3 + \sin 2t \right\}$$

$$= e^{-0} \left\{ 0^2 + 0^3 + \sin 2 \cdot 0 \right\}$$

$$= e^0 \{0\}$$

$$= 0$$

Final value theorem verification for s-domain.

$$e^{-Ts} \text{ where } T=2$$

$$f(s) = \frac{1}{(s-1)^3} + \frac{6}{(s-1)^4} + \frac{2}{(s-1)^2 + 2^2}$$

$$\frac{1}{(s-1)^3}$$

RHS

$$\lim_{s \rightarrow \infty} s f(s)$$

$$s f(s) = \frac{2s}{(s-1)^3} + \frac{6s}{(s-1)^4} + \frac{2s}{(s-1)^2 + 2^2}$$

$$s f(s) = \frac{2s}{s^3(1-1/s)^3} + \frac{6s}{s^4(1-1/s)^4} + \frac{2s}{s^2 \left\{ (1-1/s)^2 + (2/s)^2 \right\}}$$

$$= \frac{2/s^2}{(1-1/s)^3} + \frac{6/s^3}{(1-1/s)^4} + \frac{2/s}{(1-1/s)^2 + (2/s)^2}$$

$$V_{\infty} = 0$$

$$\lim_{s \rightarrow \infty} s f(s) = \frac{2}{\infty} + \frac{6}{\infty} + \frac{2}{\infty} = 0$$

$$\lim_{s \rightarrow \infty} s f(s) = 0$$

In Laplace domain a function is given by.

$$f(s) = \frac{(s-\alpha) \sin \theta}{(s^2 + \alpha^2 + \beta^2)}$$

$$f(t) = e^{-\alpha t} \cos \beta t + e^{-\alpha t} \sin \beta t$$

Final value theorem

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} s f(s)$$

$$\text{LHS} = e^{-\alpha \cdot \infty} + e^{-\alpha \cdot \infty} + e^{-\alpha \cdot \infty} \sin \infty$$

$$\text{LHS} = 0$$

$$\lim_{t \rightarrow \infty} f(t) = 0$$

RLS

$$S F(s) = \frac{25}{(s-1)^3} + \frac{65}{(s-1)^4} + \frac{25}{(s-1)^4 + 1}$$

$$S F(s) = \frac{25}{s^3(1-1/s)^3} + \frac{65}{s^4(1-1/s)^4} + \frac{25}{s^2(1-1/s)^2 + (2/s)^2}$$

$$= \frac{2/s^2}{(1-1/s)^3} + \frac{6/s^3}{(1-1/s)^4} + \frac{2/s}{(1-1/s)^2 + (2/s)^2}$$

At

$$\lim_{s \rightarrow \infty} S F(s) = \frac{2/s^2}{(1-1/s)^3} + \frac{6/s^3}{(1-1/s)^4} + \frac{2/s}{(1-1/s)^2 + (2/s)^2}$$

$$\lim_{s \rightarrow \infty} S F(s) = 0$$

4)

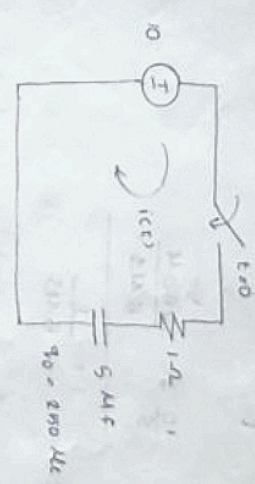
in laplace domain a function is given by

$$F(s) = M \left[\frac{(s+\alpha) \sin \theta}{(s^2 + \alpha^2 + \beta^2)} + \frac{\beta \cos \theta}{(s^2 + \alpha^2 + \beta^2)} \right]$$

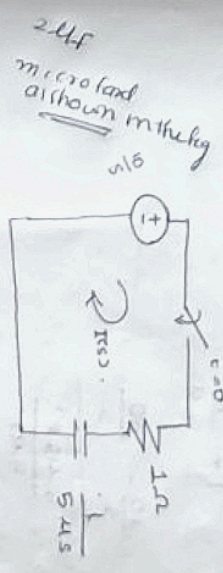
Show by initial value theorem

$$\lim_{t \rightarrow 0} f(t) = M \sin \theta$$

find $i(t)$ switching at $t=0$ assume initial charge on capacitor 250 μC (micro farads) as shown in the fig.



Laplace form of the circuit



apply KVL on the loop

$$10 u(t) - R i(t) - \frac{1}{C} \int i(t) dt = 0$$

$$10 u(t) = R i(t) + \frac{1}{C} \int i(t) dt = 0$$

$$\frac{10}{s} = R I(s) + \frac{1}{s} \left[\frac{I(s)}{s} + \frac{I(0)}{s} \right]$$

$$\frac{10}{s} = R I(s) + \left[\frac{I(s)}{s} + \frac{I(0)}{s} \right]$$

$$\frac{10}{s} = R I(s) + \left[\frac{I(s)}{s} + \frac{Q_0}{Cs} \right]$$

$$\frac{10}{s} - \frac{Q_0}{Cs} = R I(s) + \left[\frac{I(s)}{s} \right]$$

$$\frac{10}{s} - \frac{Q_0}{Cs} = I(s) \left[R + \frac{1}{s} \right]$$

$$I(s) = \left[\frac{10}{s} - \frac{90}{s} \right] \left[R + \frac{1}{s} \right]$$

$$\frac{10}{s} - \frac{2500}{54s}$$

$$1 + \frac{1}{54s}$$

$$\frac{10}{s} - \frac{50}{s}$$

$$1 + \frac{1}{54s}$$

$$\frac{10}{s} - \frac{90}{s}$$

$$\frac{54s+1}{54s}$$

$$\frac{10-90}{s}$$

$$\frac{54s+1}{54s}$$

$$\frac{-40}{s} \cdot \frac{54s}{54s+1}$$

$$\frac{-40}{s} \cdot \frac{54s}{54(s + \frac{1}{54})}$$

$$I(s) = \frac{-40 \times 54}{54(s + \frac{1}{54})}$$

$$I(s) = -40 \times \frac{1}{(s + \frac{1}{54})}$$

Taking initial laplace transform.

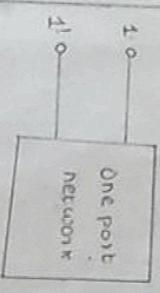
$$i(t) = -40 e^{-\frac{1}{54}t}$$

Module-4

Terminal pairs / ports

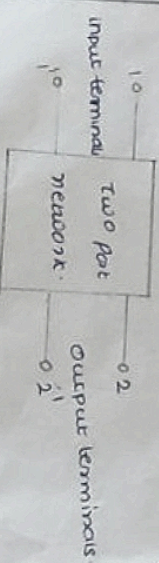
- Generally any network may be represented systematically by a rectangular box
- A pair of terminals at which a single signal may enter or leave a network is called a port

One port network



1,1' → pair of terminal - port

Two port network



1,1' → 1st port
2,2' → 2nd port

$$V \in I \in$$

$$Z = \frac{V}{I}$$

$$Z_{11} = \frac{V_1}{I_1}$$

$$Z_{22} = \frac{V_2}{I_2}$$

$$Y_{11} = \frac{I_1}{V_1}$$

$$Y_{22} = \frac{I_2}{V_2}$$

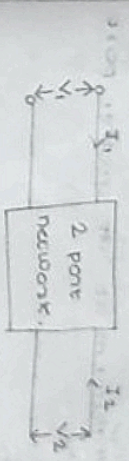
$$Z_{21} = \frac{V_2}{I_1}$$

$$Z_{12} = \frac{V_1}{I_2}$$

$$Y_{12} = \frac{I_1}{V_2}$$

$$Y_{21} = \frac{I_2}{V_1}$$

Network Functions



$$\frac{1}{Z} = Y = \text{admittance}$$

Basic equations are

$V = IZ$ Generally

$Z \rightarrow$ impedance

consider

Port 1

$$V_1 = Z_{11} I_1$$

$$Z_{11} = \frac{V_1}{I_1}$$

Port 2

$$V_2 = Z_{22} I_2$$

$$Z_{22} = \frac{V_2}{I_2}$$

Z_{11} and Z_{22} are known as driving point impedances at port 1 and port 2.

Similarly $Y_{11} = \frac{I_1}{V_1} = \frac{1}{Z_{11}}$

and $Y_{22} = \frac{I_2}{V_2}$

Here Y_{11} & Y_{22} are driving point admittances at port 1 and port 2.

$$O = \sqrt{Z_{12} Z_{21}}$$

$$O = \sqrt{Z_{12} Z_{21}}$$

$$Z_{11} = \frac{V_1}{I_1}$$

Reciprocity -
 $Z_{12} = Z_{21}$
 $Y_{12} = Y_{21}$

Voltage transfer Ratio / Voltage gain ✓

This is the ratio of the voltage transform at one port to voltage transform at another port. and is denoted by G_v

$$G_{v1} = \frac{V_2}{V_1}$$

$$G_{v2} = \frac{V_1}{V_2}$$

Current transfer Ratio / Current gain ✓

This is the ratio of the current transform at one port to current transform at other port and is denoted by G_i

$$G_{i1} = \frac{I_1}{I_2}$$

$$G_{i2} = \frac{I_2}{I_1}$$

Transfer impedance ✓

It is defined as the ratio of voltage transform at one port to the current transform at the other port and is denoted by Z

$$Z_{21} = \frac{V_2}{I_1}$$

$$Z_{12} = \frac{V_1}{I_2}$$

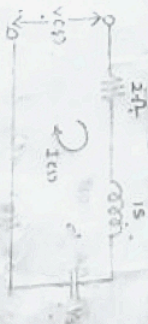
Transfer Admittance ✓

It is defined as the ratio of current transform at one port to the voltage transform at the other port and is denoted by Y

$$Y_{21} = \frac{I_2}{V_1}$$

$$Y_{12} = \frac{I_1}{V_2}$$

Obtain the driving point impedance.



Apply KVL on the

$$V(s) - 2I(s) - 15I(s) - V(s) - V(s) = 0$$

$$V(s) - 2I(s) - 15I(s) - V(s) = 0$$

$$V(s) = \left[2 - 15 \right] I(s)$$

$$V(s) = \left[2 + s + \frac{1}{s} \right] I(s)$$

$$\frac{V(s)}{I(s)} = \left[2 + s + \frac{1}{s} \right]$$

$$\frac{V(s)}{I(s)} = \frac{2s + s^2 + 1}{s}$$

$$\frac{V}{I} = Z_{11}$$

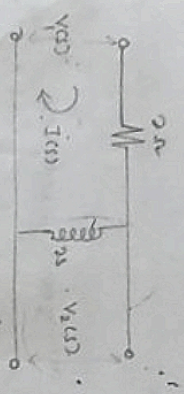
$$Z_{11} = \frac{2s + s^2 + 1}{s}$$

$$X_{11} = \frac{1}{Z_{11}}$$

$$= \frac{1}{\frac{2s+s^2+1}{s}}$$

$$= \frac{s}{2s+s^2+1}$$

obtain G_{21} & Z_{21} and Z_{11}



Apply KVL on the loop

$$V_1(s) - 2I_1(s) - \frac{2s}{s^2+1}I_1(s) = 0$$

$$V_1(s) = 2I_1(s) + \frac{2s}{s^2+1}I_1(s)$$

$$V_1(s) = (2 + \frac{2s}{s^2+1})I_1(s) \quad \text{--- (1)}$$

$$\frac{V_1(s)}{I_1(s)} = 2 + \frac{2s}{s^2+1}$$

$$Z_{11} = 2 + \frac{2s}{s^2+1}$$

Consider the output voltage.

$$V_2(s) = 2(s)I_1(s) \quad \text{--- (2)}$$

$$\frac{(2)}{(1)} = \frac{V_2}{V_1} = \frac{2(s)I_1(s)}{2 + \frac{2s}{s^2+1}I_1(s)} = G_{21}$$

$$= G_{21}$$

from (2)

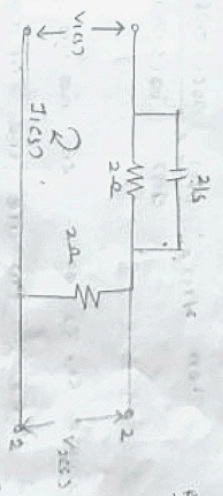
$$Z_{21} = \frac{V_2}{I_1} = \frac{V_2(s)}{I_1(s)} = \frac{2(s)I_1(s)}{I_1(s)}$$

$$= 2(s) = Z_{21}$$

from (1)

$$\frac{V_1(s)}{I_1(s)} = 2 + 2s = Z_{11}$$

Obtain G_{21} , Z_{21} , and Z_{11}



$$V_1(s) = \frac{2s}{s^2+1}I_1(s) + 2I_1(s)$$

$$= \frac{2}{s^2+1}I_1(s) + 2I_1(s)$$

$$\frac{2}{s^2+1}$$

$$V_1(s) = \frac{2}{s^2+1}I_1(s) + 2I_1(s) \quad \left[\frac{2}{s^2+1} \right] I_1(s) \rightarrow (1)$$

$$V_2(s) = 2I_1(s) \rightarrow (2)$$

$$G_{21} = \frac{V_2}{V_1} = \frac{2I_1(s)}{\frac{2}{s^2+1}I_1(s) + 2I_1(s)} = \frac{2}{2 + \frac{2}{s^2+1}} = \frac{2(s^2+1)}{2s^2+2+1} = \frac{2(s^2+1)}{2s^2+3}$$

$$Z_{21} = \frac{V_2}{I_1} = \frac{2I_1(s)}{I_1(s)} = 2$$

$$Z_{11} = \frac{V_1}{I_1} = \frac{2}{s^2+1} + 2$$

POLES and ZEROS

consider a network function

$$N(s) = \frac{P(s)}{Q(s)} = \frac{a_0 s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n}{b_0 s^m + b_1 s^{m-1} + \dots + b_{m-1} s + b_m}$$

$a_0, a_1, \dots, a_n, b_0, b_1, \dots, b_m$ etc are the coefficients of

the polynomial $P(s)$ & $Q(s)$,

They are real and positive

If the numerator and denominator of the polynomial

poles are factorised

$$\text{Then } N(s) = \frac{P(s)}{Q(s)} = \frac{a_0 (s-z_1)(s-z_2) \dots (s-z_n)}{b_0 (s-p_1)(s-p_2) \dots (s-p_m)}$$

→ where $z_1, z_2, \dots, z_n$ are the n roots for $P(s)=0$

and $p_1, p_2, \dots, p_m$ are the m roots for $Q(s)=0$

and $\frac{a_0}{b_0} = H = \text{Scaling factor}$

eg 8

$$\text{consider } N(s) = \frac{(s+1)^2 (s+3)}{(s+2)(s+2j)(s+3-2j)}$$

Zeros = No. of zeros

$$= 3$$

Zeros = $-1, -1, -3$

$$s+1=0$$

$$s=-1$$

No. of poles = $-2, -3-2j, -3+2j$

$$(s+2)^2 = -11$$

$$s+2 \Rightarrow s = -2$$

$$s+3-2j = -3-2j$$

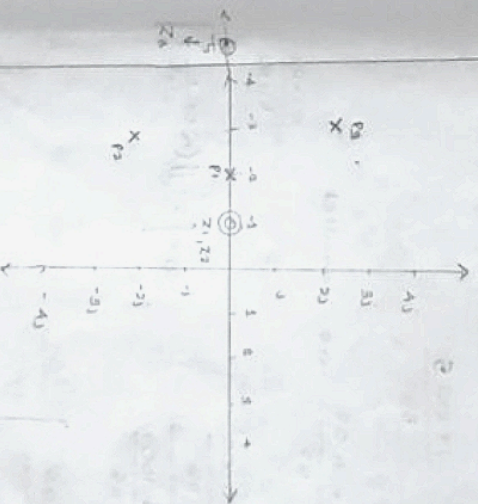
$$\text{Zeros} = \underbrace{-1, -1}_{z_1, z_2}, \underbrace{-3}_{z_3}$$

$$\text{Poles} = \underbrace{-2, -3-2j, -3+2j}_{p_1, p_2, p_3}$$

$$s = \sigma + j\omega$$

$$z = \sigma$$

$$p = \sigma$$



Poles-zero plot

determine the transfer impedance $Z(s)$ and draw the poles-zero plot

Solman

$R \rightarrow R$

$L \rightarrow LS$

$C \rightarrow \frac{1}{CS}$

$\frac{1}{C} \rightarrow \frac{1}{CS}$

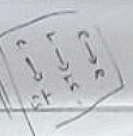
LS

$10C$

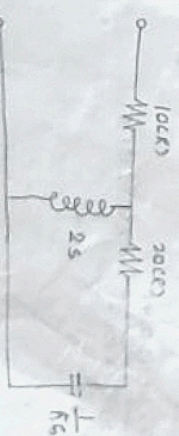
$\frac{5}{CS}$

$\frac{1}{CS}$

$C=10$



Find



$\frac{1}{h_s}$ and 20Ω are series

$$\frac{1}{h_s} + \frac{20}{1}$$

$$\frac{201}{100}$$

$$\frac{1 + 20 \times h_s}{h_s}$$

$$\frac{1 + 20h_s}{h_s} = \frac{1 + 100}{h_s}$$

25 and $\frac{1 + 100}{5s}$ are parallel

$$\frac{R_1 \times R_2}{R_1 + R_2}$$

$$Z = \frac{25 \times (1 + 100)}{5s}$$

$$\frac{25 + 1 + 100}{5s}$$

$$10 + \left[25 \parallel \left(20 + \frac{1}{5s} \right) \right]$$

$$\left[\frac{25 \times (1 + 100)}{5s} \right]$$

$$\frac{25 + 1 + 100}{5s}$$

$$= \frac{10s^2 + 100s + 1}{5s}$$

$$\frac{10s^2 + 100s + 1}{5s}$$

Z

$$\frac{10s^2 + 100s + 1}{10s^2 + 100s + 1}$$

10 & $\frac{10s^2 + 100s + 1}{10s^2 + 100s + 1}$ are series.

$$\frac{2 \times (1 + 100)}{h_s}$$

$$25 + \frac{1 + 100}{h_s}$$

$$\frac{200s + 2}{h_s}$$

$$25 + \frac{1 + 100}{5s}$$

$$\frac{200s + 2}{h_s} \times \frac{h_s}{10s^2 + 1100s}$$

10 and

$$\frac{200s + 2}{h_s} \times \frac{h_s}{10s^2 + 1100s}$$

are parallel series.

$$= 10 + \left[\frac{200s + 2}{h_s} \times \frac{h_s}{10s^2 + 1100s} \right]$$

$$= 10 + \left[\frac{200s^2 + 2s}{10s^2 + 1100s + 1} \right]$$

$$= \frac{200s^2 + 1002s + 10}{10s^2 + 1100s + 1}$$

$$\frac{10(200s^2 + 1002s + 10)}{10s^2 + 1100s + 1}$$

No. of zeros = 2.

Poles = 2

$$\sqrt{Z_1} = -0.01$$

$$P_1 = -0.01$$

$$Z_2 = -0.01$$

$$P_2 = -9.98$$

$$-3.8$$

Time domain response from pole-zero plot

Step 1) Obtain the pole-zero plot for the given transfer function

Step 2) Measure the distances $M_{za}, M_{zb}, \dots, M_{zn}$ of the given pole from each of the other zeros.

Step 3) Measure the distances $M_{p1}, M_{p2}, \dots, M_{pm}$ of a given pole from each of the other poles.

Step 4) measure the angle $\phi_{za}, \phi_{zb}, \dots, \phi_{zn}$ of the line joining that pole to each of the other zeros.

Step 5) measure the angle $\phi_{p1}, \phi_{p2}, \dots, \phi_{pm}$ of the line joining that pole to each of the other poles.

Step 6) substitute these values in the equation.

$$K_1 = H \frac{M_{za} M_{zb} \dots M_{zn}}{M_{p1} M_{p2} \dots M_{pm}} \times e^{j[\phi_{za} + \phi_{zb} + \dots + \phi_{zn}] - [\phi_{p1} + \phi_{p2} + \dots + \phi_{pm}]}$$

For the pole-zero diagram for the given network work function and hence obtain $\uparrow$ i.e.

$$I(s) = \frac{205}{(s+1)(s+2)}$$

$$1(s) = ?$$

$$Zeros = x_1 = 0.$$

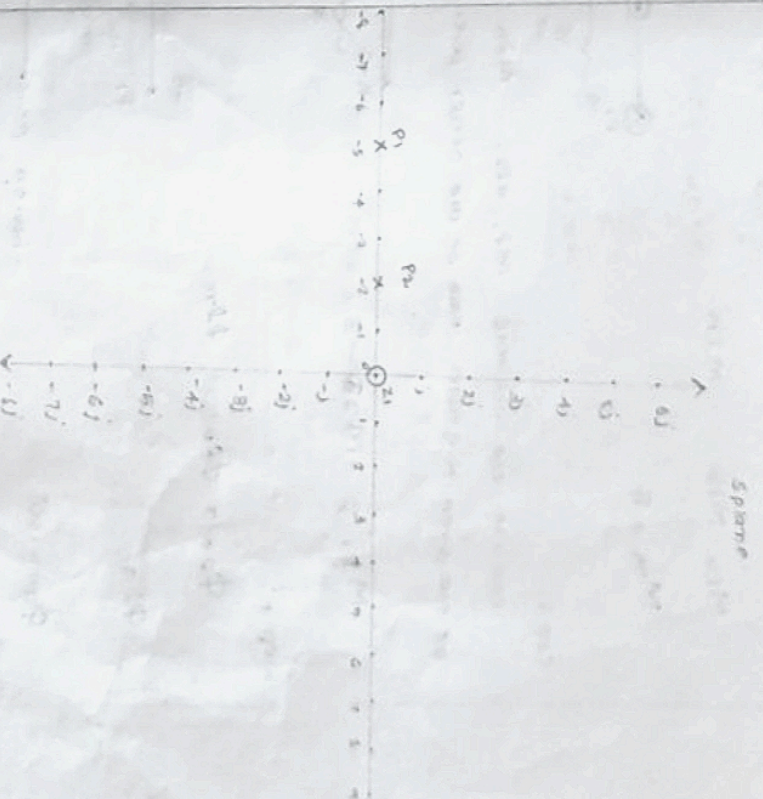
$$\text{Here } H = \text{Scaling} = 205.$$

$$\text{no. of zeros} = 1.$$

$$\text{no. of poles} = 2.$$

$$\text{Poles are } -5 \text{ \& } -2.$$

Step 1



$$\text{Step 2: } I(s) = \frac{205}{(s+1)(s+2)}$$

Taking for inverse Laplace transform.

$$I(s) = \frac{K_1}{s+1} + \frac{K_2}{s+2}$$

$$= K_1 e^{-6t} + K_2 e^{-2t}$$

forgetting k_1, k_2

step 2: measure the distance $M_{1a}, M_{1b}, \dots, M_{1n}$ of the given poles from each of the other zeros.

$$M_{1a}, M_{1b}, \dots, M_{1n}$$

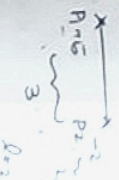
$$M_{05} = 5$$



step 3:

measure the distance $M_{21}, M_{22}, \dots, M_{2n}$ of the given pole from each of the other poles.

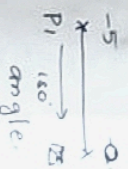
$$M_{21} = M_{25} = 3$$



step 4:

$$\phi_{2a}, \phi_{2b}, \dots, \phi_{2n}$$

$$\phi_{2a}$$



$$\phi_{05} = 180^\circ$$

Chosen points $\rightarrow$ right side

step 5:

$$\phi_{21} = 180^\circ$$

left side $\leftarrow$ Chosen points 0°

$$K_1 = \frac{1}{M_{21}} \cdot \frac{M_{1a}}{e^{j(\phi_{2a} - \phi_{1b})}} \cdot \frac{P_1}{P_2} \cdot \frac{1}{e^{j(180 - 180)}} \cdot \frac{1}{180^\circ}$$

$$K_1 = \frac{1}{3} \cdot \frac{20}{5} \cdot \frac{1}{e^{j(180 - 180)}} \cdot \frac{1}{180^\circ}$$

$$K_1 = 20 \times \frac{1}{3} e^{j(0^\circ)}$$

$$K_2 = 20 \times \frac{1}{3} e^{j(0^\circ)}$$

$$K_2 = \frac{20 \times 5}{3} = \frac{100}{3} = 33.3$$

$$K_2 = \frac{20 \times 5}{3} = \frac{100}{3} = 33.3$$

$$K_2 = \frac{20 \times 5}{3} = \frac{100}{3} = 33.3$$

pole 2

step 2:

$$M_{2a}, M_{2b}, \dots, M_{2n}$$

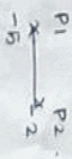
$$M_{2a} = M_{02} = 2$$



step 3:

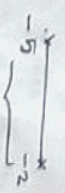
$$M_{21}, M_{22}, \dots, M_{2n}$$

$$M_{21} = M_{22} = 3$$



step 4: ϕ_{2a}

$$\phi_{22} = 180^\circ$$



step 5:

$$\phi_{21}$$

$$\phi_{22} = 0^\circ$$

$$K_1 = \frac{1}{M_{21}} \cdot \frac{M_{2a}}{e^{j(\phi_{2a} - \phi_{1b})}} \cdot \frac{P_1}{P_2} \cdot \frac{1}{e^{j(180 - 0)}} \cdot \frac{1}{180^\circ}$$

$$K_1 = \frac{1}{3} \cdot \frac{20}{5} \cdot \frac{1}{e^{j(180 - 0)}} \cdot \frac{1}{180^\circ}$$

$$e^{j\omega t} = \cos \omega t + j \sin \omega t$$

$$= \cos 180 + j \sin 180$$

$$= -1 + j0$$

$$K_2 = 1 \cdot \frac{8}{3} \times e^{j/180} = \frac{-1}{3}$$

$$K_1 = 20 \times \frac{2}{3} \times (-1)$$

$$\frac{10 \times -1}{3} = -1 +$$

$$\frac{-10}{3} = -3.33$$

$$K_1 e^{-5t} + K_2 e^{-3t}$$

$$33.3 e^{-5t} + 13.33 e^{-3t}$$

$$33.3 e^{-5t} - 13.33 e^{-3t}$$

H(s)

$$V(s) = \frac{4.5(s+2)}{(s+1)(s+3)}$$

$$\text{Zeros} = 2, 1, 2, 2$$

Here $H = + \Rightarrow$ scaling factor?

No. of zeros = 2

$$Z_1 = 0 \quad Z_2 = 2$$

Poles = P_1, P_2

No. of poles = 2

$$P = -1 \quad P = -3$$



for getting K_1 & K_2

consider pole at -3

$$M_{03} = 3 \quad M_{23} = 1$$

$$\phi_{03} = 180^\circ \quad \phi_{23} = 180^\circ$$

$$M_{13} = 2$$

$$\phi_{13} = 180^\circ$$

consider pole at -1

$$M_{01} = 1 \quad M_{21} = 1$$

$$\phi_{01} = 180^\circ \quad \phi_{21} = 0^\circ$$

$$M_{31} = 2$$

$$\phi_{31} = 0$$

$$K_2 = 1 \cdot \frac{M_{01} M_{21}}{M_{31}} \times e^{j(\phi_{01} + \phi_{21}) - (\phi_{31})}$$

$$= \frac{4 \times 1 \times 1}{2} \times e^{j(180+0) - (0)}$$

$$= 2 e^{j180}$$

$$K = -2$$

$$V(s) = -2 e^{-t} - 6 e^{-3t}$$

2

$$I(s) = \frac{Ks}{(s+1)(s^2+4s+8)}$$

SOLN:
 $I(s) = \frac{Ks}{(s+1)(s^2+4s+8)}$

No. of zeros = 1

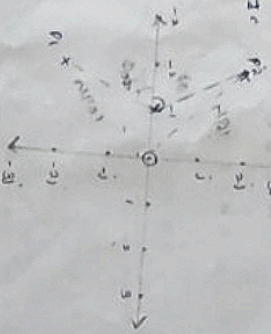
$z_1 = 0$

No. of poles = 3

$$P_1 = -1 \quad P_2 = -2+2j \quad P_3 = -2-2j$$

$$K(s) = \frac{Ks}{(s+1)(s-(-2+2j))(s-(-2-2j))}$$

$$I(s) = \frac{K_1}{s+1} + \frac{K_2}{s-(-2+2j)} + \frac{K_3}{s-(-2-2j)}$$



consider P_1
 $M_{1a} \quad M_{1b} \dots \quad M_{1n}$

$$\rightarrow M_{01} = 1$$

$$\phi_{01} = 180^\circ$$

$$M_{P_2} = M_{P_3} = \sqrt{h}$$

$$\phi_{P_2} = -63^\circ$$

$$\phi_{P_3} = 63^\circ$$

$$\frac{s^2+4s+8}{2s} = \frac{-4 \pm \sqrt{16-4 \times 1 \times 8}}{2 \times 1}$$

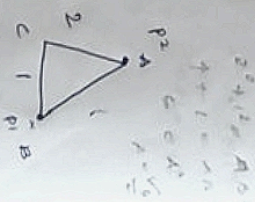
$$= \frac{-4 \pm \sqrt{16-32}}{2}$$

$$= \frac{-4 \pm \sqrt{-16}}{2}$$

$$= \frac{-4 \pm \sqrt{-16}}{2}$$

$$= \frac{-4 \pm \sqrt{-16}}{2}$$

$$(s+1)(s+2 \pm 2j)$$



$$AB^2 = 2^2 + 1^2$$

$$AB^2 = 4+1$$

$$AB^2 = 5$$

$$AB = \sqrt{5}$$

point -1

M_{1a}

consider P_2

$$M_{0P_2} = \sqrt{8}$$

right angle

apply Pythagorean theorem

$$\sqrt{2^2+2^2} = \sqrt{4+4} = \sqrt{8}$$

$$\phi_{0P_2} =$$

$$M_{P_3P_2} = 4$$

$$M_{P_1P_2} = \sqrt{h}$$

$$\phi_{0P_2} = 135^\circ$$

$$\phi_{P_3P_2} = 90^\circ$$

$$\phi_{P_1P_2} = 115^\circ$$

$$K_2 = \frac{M_{0P_2}}{M_{P_3P_2} M_{P_1P_2}} e^{j(\phi_{0P_2} - \phi_{P_3P_2} - \phi_{P_1P_2})}$$

$$K_2 = \frac{4 \times \sqrt{5}}{4 \times \sqrt{5}} e^{-j(135^\circ - 90^\circ - 115^\circ)}$$

$$K_2 = \frac{4 \times \sqrt{5}}{4 \times \sqrt{5}} e^{-j11^\circ}$$

$$K_2 = \frac{4 \times \sqrt{5}}{4 \times \sqrt{5}} e^{-j11^\circ}$$

$$= 1.58 e^{-j11^\circ}$$

$P_2 = a+ib$
 $P_3 = a-ib$ } conjugate

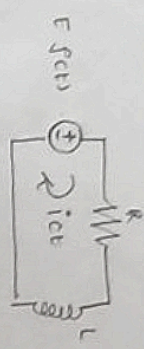
consider P_3 here P_2 & P_3 are conjugates

$$K_3 = 1$$

$$K_3 = K_2 = 1.58 e^{-j11^\circ}$$

31/10/2022
Monday

Exam Review Impulse response of RL series circuit



$$E f(t) - R i(t) - L \frac{di(t)}{dt} = 0$$

$$E f(t) = R i(t) + L \frac{di(t)}{dt}$$

→ Taking Laplace transform

$$E(s) = R I(s) + L [s I(s) - i(0)]$$

$$E = R I(s) + L [s I(s)]$$

$$E = R I(s) + L s I(s)$$

$$E = I(s) [R + Ls]$$

$$I(s) = \frac{E}{[R + Ls]} L$$

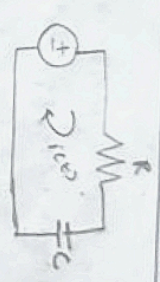
$$I(s) = \frac{E}{L} \left[\frac{1}{s + \frac{R}{L}} \right]$$

$$I(s) = \frac{E}{L} \left[\frac{1}{s + \frac{R}{L}} \right]$$

$$i(t) = \frac{E}{L} e^{-R/L t}$$

Sub in FL

Impulse response of RC series circuit



$$E f(t) - R i(t) - \frac{1}{C} \int i(t) dt = 0$$

$$E f(t) = R i(t) + \frac{1}{C} \int i(t) dt$$

Taking Laplace transform

$$E(s) = R I(s) + \frac{1}{C} \left[\frac{I(s)}{s} - \frac{i(0)}{s} \right]$$

$$E = R I(s) + \frac{I(s)}{Cs}$$

$$E = I(s) \left[R + \frac{1}{Cs} \right]$$

$$I(s) = \frac{E}{R + \frac{1}{Cs}}$$

$$I(s) = \frac{E}{\frac{R Cs + 1}{Cs}}$$

$$I(s) = \frac{E Cs}{R Cs + 1}$$

$$I(s) = \frac{E Cs}{R Cs + 1}$$

$$I(s) = \frac{E}{R} \left[\frac{s}{s + \frac{1}{RC}} \right]$$

$$I(s) = \frac{E}{R} \left[\frac{s + \frac{1}{RC} - \frac{1}{RC}}{s + \frac{1}{RC}} \right]$$

$$i(t) = \frac{E}{R} \left[1 - e^{-t/RC} \right]$$

Tuesday
2/10/2020

Lab 2

$V = e^{-2x} \sin y$ is harmonic and hence find the conjugate function

$$V = e^{-2x}$$

$$V_x = \frac{\partial}{\partial x} e^{-2x} = -2e^{-2x}$$

$$V_{xx} = \frac{\partial}{\partial x} (-2e^{-2x}) = 4e^{-2x}$$

$$4e^{-2x}$$

$$V_{xx} = \frac{\partial^2}{\partial x^2} e^{-2x} = 4e^{-2x}$$

$$V_x = -2e^{-2x} \sin y$$

$$V_y = 2e^{-2x} \cos y$$

apply CR eqn:

$$f'(z) = U_x + iV_x$$

$$V_y + iV_x$$

$$2e^{-2x} + i(-2e^{-2x} \sin y)$$

$$2e^{-2x} + i(-2e^{-2x} \sin y)$$

$f'(z) = 2e^{-2x}$ integrating on both side

$$f(z) = \int 2e^{-2x} dx$$

$$= -2 \int e^{-2x} dx$$

$$= -2 \left(\frac{e^{-2x}}{-2} \right)$$

$$f(z) = -e^{-2x}$$

$$= -e^{-2x} + i y$$

SPH:

$$f(z) = -e^{-2x + iy}$$

$$= -e^{-2x - i y}$$

$$= -e^{-2x} e^{-i y}$$

$$= e^{-2x} [\cos y - i \sin y]$$

$$= \frac{e^{-2x} (\cos y + i \sin y) + e^{-2x} (\cos y - i \sin y)}{2}$$

$$V = e^{-2x} \cos y \quad U = e^{-2x} \sin y$$

2/10/2020 - 15:00